Applications

- a. As distance increases, weight decreases. The decrease is sharper at shorter distances. (The product of distance and weight is always 90,000.)
 - **b.** The graph shows that as distance increases, weight decreases—sharply at first, and then more gradually.



- **c.** 5,000 lb; ≈ 3,000 lb; ≈ 1,250 lb
- **d.** The graph's shape is similar to that of the bridge-length experiment because the values of the dependent variable decrease at a decreasing rate.



Figure 1

The data are very close to linear. Each time the class adds two layers, the bridge can hold approximately 15 more pennies.

- b. 28 pennies. The breaking weight is about 8 pennies per layer. So, for 3.5 layers, the breaking weight would be 28.
- c. 80. The breaking weight is about 8 pennies per layer. So, for 10 layers, the breaking weight would be 80.
- **3. a.** (See Figure 1.)



c. This is a linear relationship. As truss length increases by 1 unit, cost increases by \$9.

Cost of CSP Trusses											
Truss Length (ft)	1	2	3	4	5	6	7	8			
Number of Rods	3	7	11	15	19	23	27	31			
Cost of Truss	\$56.75	\$65.75	\$74.75	\$83.75	\$92.75	\$101.75	\$110.75	\$119.75			

d. (See Figure 2.)



2

0

f. This is not a linear relationship. As the number of steps increases by 1, the cost increases at an increasing rate.

4

Number of Steps

6

8

4. a. (See Figure 3.)



c. This is not a linear relationship. In the table, when you add the second medal winner, you add 2 boxes. When you add a third medal winner, you add 3 more boxes. To add a 29th medal winner, you add 29 boxes to a 28-step platform. The change is increasing at each step. You see this in the graph because the graph rises more and more sharply as you move from left to right along the x-axis.

Figure 2

Costs of CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	28	40	54	70	88
Cost of Frame	\$59	\$72.50	\$90.50	\$113	\$140	\$171.50	\$207.50	\$248

Figure 3

Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	10	15	21	28	36

d. (See Figure 4.)



- **f.** The pattern in the points illustrates a linear relationship because, with every new step, the length of the red carpet increases by exactly 3 feet. This constant rate of change is different than the pattern in the number of boxes, which has an increasing rate of change.
- 5. a. linear
 - **b.** nonlinear

Note: Students may find this problem tricky because it does not make sense to make stairs with 1 rod, or 2 or 3, or some of the other choices they may see if they make a table relating number of rods to cost of rods.

Figure 4

Carpet for Platforms

Number of Steps	1	2	3	4	5	6	7	8
Carpet Length (ft)	13	16	19	22	25	28	31	34

Figure 5

CSP Ladder Bridges

Bridge Length (ft)	1	2	3	4	5	6
Number of Rods	4	7	10	13	16	19

- c. linear
- d. nonlinear
- e. linear
- f. nonlinear
- **g.** The relations in parts (b) and (f) are increasing, but at different rates. The relationship in part (d) is decreasing.
- **6. a.** (See Figure 5.)



b. This is an increasing linear relationship like the relationship between truss length and number of rods. Although the relationship between number of steps and number of rods in a staircase frame is also increasing, it is not linear.

Thinking With Mathematical Models

Connections



- **b.** 300 ft; 150 ft; 100 ft
- c. $\frac{300}{\ell}$ ft. Note: Some students may not be able to use symbols to describe this relationship. They will work more with the relationships among area, length, and width in Investigation 3.
- d. The width decreases, but not linearly.
- **e.** The graph decreases very sharply at first and then more gradually.





- **b.** 34 ft; 33 ft; 35ℓ ft, or $0.5(70 2\ell)$ ft
- **c.** 34.5 ft, 33.5 ft
- **d.** 15 ft by 20 ft; about 15.6 ft by 19.4 ft; 17.5 ft by 17.5 ft
- e. It decreases linearly.





f. There is a linear decrease in the graph.

- 11. (See Figure 6.)
 - **a.** Use the "Probable Sales" row in the table.
 - **b.** Use the "Probable Income" row in the table.
 - **c.** \$2.50
- Answers will vary according to students' choice of babysitting rates.
 - **a.** Use a rate of \$5 per hour, and mark axes for perhaps 20 hours and \$100.
 - **b.** y = 5x
 - **c.** Jake would earn more per hour, for example, \$6 per hour. The equation would then be y = 6x.

13. a. *x* = −2







c. $x = -3\frac{1}{3}$



- **d.** In each case, the solution of the first equation x is the x-intercept of the second equation. This makes sense because in each case the first equation is the same as the second equation except that y has been replaced by 0. So the first equation can be solved by looking at the corresponding graph and asking, "What will the answer be for x when y = 0? Or, what is the value of x at the point (x, 0)?"
- 14. Graph C
- 15. Graph A
- **16.** Graph D
- 17. Graph B
- **18.** 2 coins. Possible method: Take 3 coins from each side to find 3 pouches equals 6 coins. Because each pouch contains the same number of coins, there must be 2 coins in each pouch.

Figure 6

Predicted Ticket Sales for Whole School

Ticket Price	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00	\$3.50	\$4.00	\$4.50
Probable Sales	400	400	360	300	240	200	160	140
Probable Income	\$400	\$600	\$720	\$750	\$720	\$700	\$640	\$630

- 19. 3 coins. Possible method: Take 1 coin from each side to find 4 pouches equals 2 pouches and 6 coins. Now take 2 pouches from each side to find 2 pouches equals 6 coins. Because each pouch contains the same number of coins, there must be 3 coins in each pouch.
- **20.** a. 3x + 3 = 9 and 4x + 1 = 2x + 7
 - **b.** Possible solution for 3x + 3 = 9:

3x + 3 = 9

- 3x = 6 Subtract 3 from each side.
- x = 2 Divide each side by 3.

Possible solution for 4x + 1 = 2x + 7:

4x + 1 = 2x + 7

4x = 2x + 6 Subtract 1 from each side.

- 2x = 6 Subtract 2x from each side.
 - x = 3 Divide each side by 2.
- **c.** The strategies were the same, but in part (b) symbols were used instead of objects.
- **21.** *x* = 2
- **22.** *x* = 4
- **23.** $x = \frac{14}{6}$ or an equivalent form **24.** $x = -\frac{2}{6}$

24.
$$x = -\frac{2}{3}$$

- **25.** $x = 2\frac{1}{8}$ or x = 2.125
- **26.** *x* = −2
- **27.** *x* = −3
- **28.** *x* = 4
- **29.** false, because 42 < 50
- **30.** true, because 11 > 6
- **31.** false, because -10 < 0
- **32.** a. The "wrap" part of the cylinder has the same area $(8.5 \times 11 \text{ sq in.})$ for each cylinder. But the circular bases are larger for the cylinder with the 8.5-inch height.
 - **b.** See part (a) above.
 - c. See part (a) above.
 - **d.** The shorter cylinder; the base area depends on the radius. If the smaller dimension of the paper is used for the height of the cylinder then the base area will have a larger radius.
 - **33.** Answers will vary. The only criterion is that $r^2h = 28 \text{ cm}^3$. Possible answers: $r = 2 \text{ cm}, h = 7 \text{ cm}; r = \sqrt{7} \text{ cm},$ $h = 4 \text{ cm}; r = \sqrt{8} \text{ cm}, h = 3.5 \text{ cm}$

Extensions

34. a.

x	p	q	у	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	<u>1</u> 3
4	16	64	16	$\frac{1}{4}$
5	25	125	32	<u>1</u> 5
6	36	216	64	<u>1</u> 6
10	100	1,000	1,024	<u>1</u> 10
11	121	1,331	2,048	$\frac{1}{11}$
12	144	1,728	4,096	<u>1</u> 12
n	n ²	n ³	2 ⁿ	$\frac{1}{n}$

- **b.** None of the patterns are linear because a constant change in *x* does not yield a constant change in *y*.
- **35.** Modeling data patterns.
 - **a.** The three scatter plots will look like this:



CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	28	40	54	70	88

Height vs. Foot Length



- **b.** Only the (*height*, *foot length*) graph looks linear.
- **c.** Approximately 6 : 1; The average student is 6 "feet" tall.
- d. Shoshana White; Tonya Stewart

- 36. Staircase as prism.
 - a. Orientation of base will vary, but here is one possible sketch from an overhead perspective; area is 6 units². This "2" should be superscripted.



b. One possible sketch would be as follows:



Surface Area = top + bottom + left + rear + step + step + step

Surface Area = [6 + 6 + 9 + 9 + (3 + 3) + (3 + 3) + (3 + 3)] units². This "2" should be superscripted.

Surface Area = 48 units^2 . This "2" should be superscripted.

c. New Surface Area = top + bottom + left + rear + step + step + step + step + step + step

New Surface Area = [21 + 21 + 18 + 18 + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3)] units². This "2" should be superscripted.

New Surface Area = 114 units². This "2" should be superscripted. The top and bottom areas more than doubled. The left and rear areas exactly doubled (but they are no longer squares). The "stair" area doubles. So the total area is more than twice the original. A flat pattern is shown below.

(See Figure 7.)



Thinking With Mathematical Models