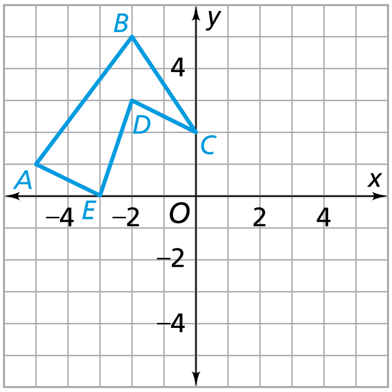
Applications

For Exercises 1–7, make a copy of the figure below. Then, find the image of the figure after each transformation.



1. Copy and complete the table showing the coordinates of points *A*–*E* and their images after a reflection in the *y*-axis.

Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(−5, 1)	(−2, 5)	■	■	■
Coordinates After a *y*-axis Reflection	■	■	■	■	■

a. Draw the image.

b. Write a rule relating coordinates of key points and their images after a reflection in the *y*-axis: $(x, y) \rightarrow (■, ■)$.

2. Add a row to your table from Exercise 1 to show the coordinates of points *A*–*E* and their images after a reflection in the *x*-axis.

a. Draw the image.

b. Write a rule relating coordinates of key points and their images after a reflection in the *x*-axis: $(x, y) \rightarrow (■, ■)$.

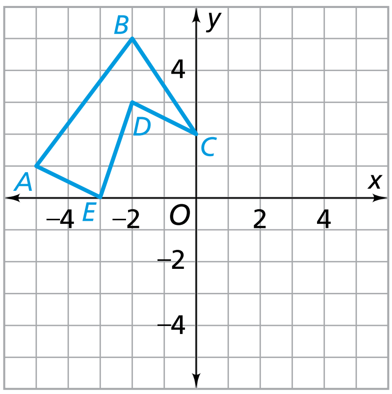
3. Add another row to your table from Exercise 1 to show the coordinates of points *A*–*E* and their images after a reflection in the *x*-axis, followed by a reflection in the *y*-axis.

a. Draw the final image.

b. Write a rule relating coordinates of key points and their images after both reflections: $(x, y) \rightarrow (■, ■)$.

c. What single transformation in this Investigation has the same effect as the sequence of two line reflections?

4. Copy and complete the table showing the coordinates of points *A*–*E* and their images after a translation that "moves" point *B* to point (3, 4).



Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(−5, 1)	(−2, 5)	■	■	■
Coordinates After Translating *B* to (3, 4)	■	■	■	■	■

a. Draw the image.

b. Write a rule relating coordinates of key points and their images after the translation: $(x, y) \rightarrow (■, ■)$.

5. Add a row to your table from Exercise 4 to show the coordinates of points A–E and their images after the first translation, followed by a translation that “moves” point B' to $(-1, 0)$.

a. Draw the image.

b. Write a rule relating coordinates of key points and their images after the second translation: $(x, y) \rightarrow$ (■, ■).

c. Write a rule relating coordinates of key points and their images after the sequence of the two translations: $(x, y) \rightarrow$ (■, ■).

d. What single transformation is equivalent to the sequence of the two translations?

6. Copy and complete the table showing the coordinates of points A–E and their images after a counterclockwise rotation of 90° about the origin.

Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(–5, 1)	(–2, 5)	■	■	■
Coordinates After a 90° Rotation	■	■	■	■	■

a. Draw the image.

b. Write a rule relating coordinates of key points and their images after a rotation of 90°: $(x, y) \rightarrow$ (■, ■).

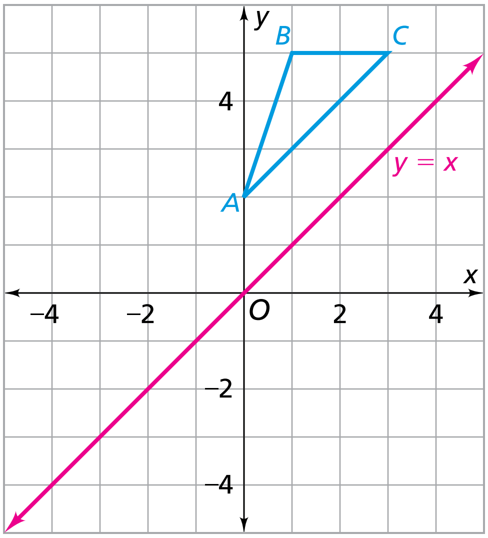
7. Add a row to your table from Exercise 6 to show the coordinates of points A–E and their images after two counterclockwise rotations of 90° about the origin.

a. Draw the final image.

b. Write a rule relating coordinates of key points and their images after both rotations: $(x, y) \rightarrow$ (■, ■).

c. What single transformation is equivalent to the sequence of the two rotations?

8. **a.** Use triangle ABC shown in the diagram.



Copy and complete the table showing the coordinates of points A–C and their images after a reflection in the line $y = x$.

Point	A	B	C
Original Coordinates	■	■	■
Coordinates After a Reflection in $y = x$	■	■	■

b. Draw the image and label the vertices A', B', and C'.

c. Add a row to your table to show the coordinates of points A–C and their images after a reflection of triangle $A'B'C'$ in the x-axis.

d. Draw the image and label the vertices A'', B'', and C''.

e. Draw the image of triangle ABC after the same two reflections, but in the reverse order. That is, reflect triangle ABC in the x-axis and then reflect its image, triangle $A'B'C'$, in the line $y = x$. What does the result suggest about the commutativity of a sequence of line reflections?

9. **a.** Use triangle *ABC* from Exercise 8. Draw triangle *ABC* on a coordinate grid.

i. Translate *ABC* according to the rule $(x, y) \rightarrow (x + 2, y - 3)$. Label its image $A'B'C'$.

ii. Translate *ABC* according to the rule $(x, y) \rightarrow (x - 4, y - 6)$. Label its image $A''B''C''$.

b. Use the coordinates of the vertices of triangle *ABC* and its two images to compare the slopes of each pair of line segments.

i. $\overline{AB}$ and $\overline{A'B'}$; $\overline{AC}$ and $\overline{A'C'}$; $\overline{CB}$ and $\overline{C'B'}$

ii. $\overline{AB}$ and $\overline{A''B''}$; $\overline{AC}$ and $\overline{A''C''}$; $\overline{CB}$ and $\overline{C''B''}$

c. What do your results from parts (a) and (b) say about the effect of translations on the slopes of lines? About the relationship between a line and its image under a translation?

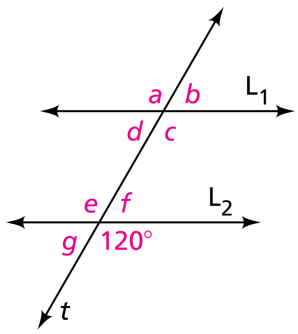
10. **a.** Use triangle *ABC* from Exercise 8. Draw triangle *ABC* on a coordinate grid and its image after a 180° rotation about the origin. Label the image $A'B'C'$.

b. Use the coordinates of the vertices of triangle *ABC* and its image to compare the slopes of each pair of line segments.

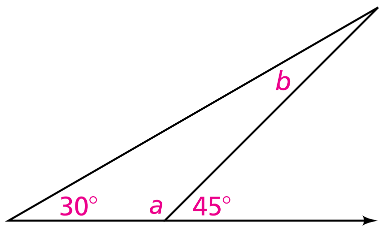
i. $\overline{AB}$ and $\overline{A'B'}$ **ii.** $\overline{AC}$ and $\overline{A'C'}$ **iii.** $\overline{CB}$ and $\overline{C'B'}$

c. What do your results from parts (a) and (b) say about the effect of half-turns or 180° rotations on the slopes of lines? About the relationship between a line and its image under a 180° rotation?

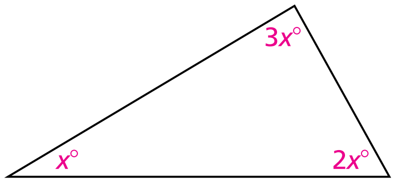
11. In the diagram below, lines L_1 and L_2 are parallel. What are the measures of angles *a*–*g*?



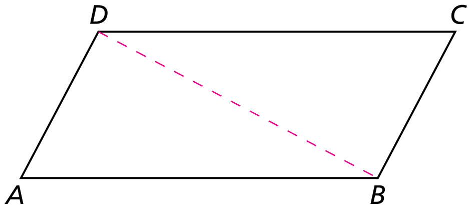
12. What are the measures of angles a and b in the triangle at the right?



13. What is the value of x in the diagram at the right?



14. The diagram at the right shows parallelogram $ABCD$ with one diagonal DB. Assuming only that opposite sides of any parallelogram are parallel:



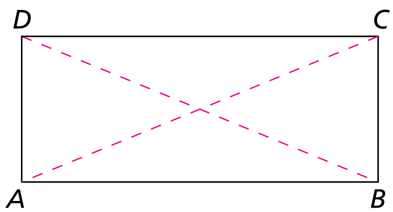
a. Which angles are congruent? How do you know?

b. How can you be sure that triangle ABD is congruent to triangle ADB? What are the corresponding vertices, sides, and angles?

c. How does the congruence of triangles ABD and ADB imply that the opposite angles of the parallelogram are congruent?

d. How does the congruence of triangles ABD and ADB guarantee that, in a parallelogram, opposite sides are the same length?

15. The diagram below shows a rectangle with two diagonals.



a. How can you be sure that triangle ABC is congruent to triangle BAD?

b. Why does this congruence guarantee that, in a rectangle, the diagonals are the same length?

Connections

16. Copy and complete the table of values for the function $y = -x^2$.
Remember: $-(-3)^2 = -9$.

x	–3	–2	–1	0	1	2	3
y	■	■	■	■	■	■	■

a. Use the table of values to graph the function $y = -x^2$.

b. Describe the symmetries of the graph of the function $y = -x^2$.

17. Add a row to your table from Exercise 16 to show values of the function $y = -x^2 + 4$.

a. Use the values in the extended table. Graph $y = -x^2 + 4$ on the same coordinate grid as $y = -x^2$ from Exercise 16.

b. Write a coordinate rule that "moves"

i. the graph of $y = -x^2$ to the position of the graph of $y = -x^2 + 4$.

ii. the graph of $y = -x^2 + 4$ to the position of the graph of $y = -x^2$.

18. Complete the table of values for the function $y = |x|$.
Remember: $|-4| = |4| = 4$.

x	–4	–3	–2	–1	0	1	2	3	4
y	■	■	■	■	■	■	■	■	■

a. Use the table of values to graph the function $y = |x|$.

b. Describe the symmetries of the graph of the function $y = |x|$.

19. Add a row to your table from Exercise 18 to show values of the function $y = |x| - 3$.

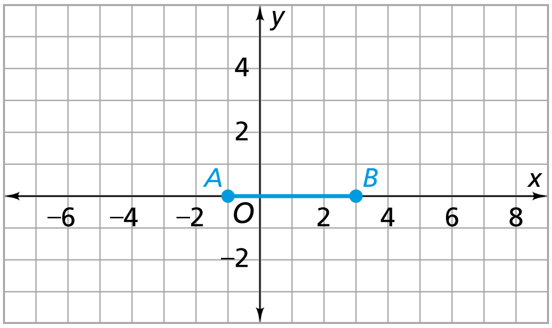
a. Use the values in the extended table. Graph $y = |x| - 3$ on the same coordinate grid as $y = |x|$ from Exercise 18.

b. Write a coordinate rule that "moves"

i. the graph of $y = |x|$ to the position of the graph of $y = |x| - 3$.

ii. the graph of $y = |x| - 3$ to the position of the graph of $y = |x|$.

20. Points A and B are on the x-axis.



a. Compare the x-coordinates of points A and B.

b. Translate points A and B five units to right. Compare the x-coordinates of the image points.

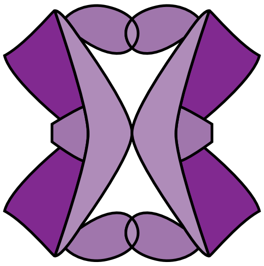
c. Translate points A and B five units to left. Compare the x-coordinates of the image points.

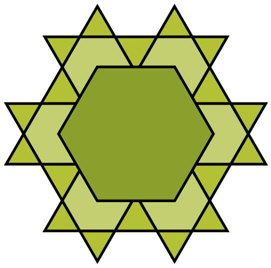
d. Rotate points A and B 180° about the origin. Compare the x-coordinates of the image points.

e. Write a general rule about the effect of adding or subtracting a constant c from two integers, a and b. Complete the following sentence: If $a < b$, then when you add a constant c to a and b . . . .

f. Write a general rule about the effect of multiplying integers a and b by -1. Complete the following sentence: If $a < b$, then when you multiply each by -1. . . .

For Exercises 21 and 22, describe the symmetries of each design.

21. 

22. 

23. Multiple Choice Squares, rectangles, and rhombuses are all types of parallelograms. Which of these statements is true for all parallelograms?

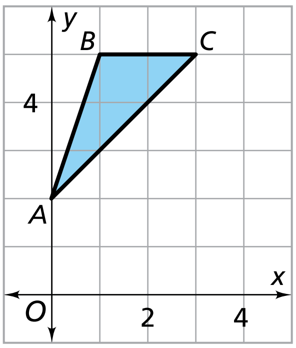
A. The diagonals are congruent.

B. Each diagonal divides the other in two congruent segments.

C. The diagonals divide a parallelogram into four congruent triangles.

D. The diagonals bisect the angles at each vertex.

24. What is the area of triangle *ABC*?

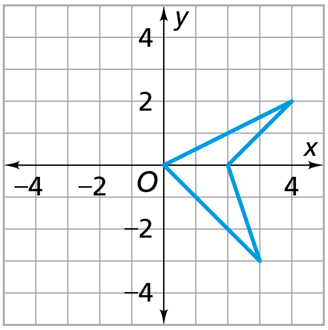


25. What are the side lengths and the perimeter of triangle *ABC* from Exercise 24?

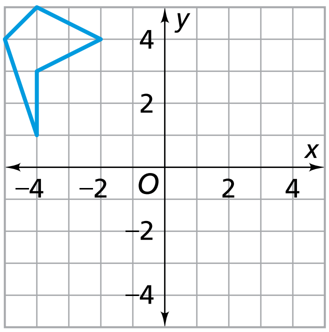
Extensions

For Exercises 26–28, draw the figure on grid paper. Then, use symmetry transformations to draw a design that meets the given condition(s). Describe the transformations you used and the order in which you applied them.

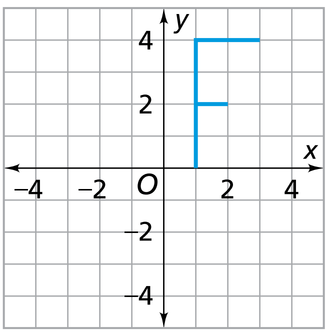
26. a design that has at least two lines of symmetry



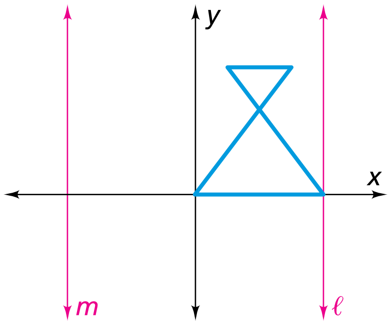
27. a design that has rotational symmetry



28. a design that has both reflectional and rotational symmetry



29. Multiple Choice Which of these statements is *not* true about the figure below if lines m and ℓ are parallel?



F. Reflecting the figure in the y-axis, and then reflecting the image in the x-axis, gives the same final image as rotating the figure 180° about the origin.

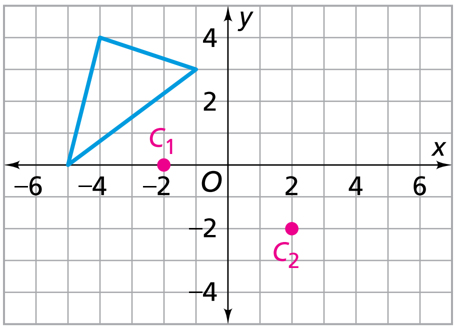
G. Reflecting the figure in line ℓ, and then reflecting the image in the y-axis, gives the same final image as reflecting the figure in line m.

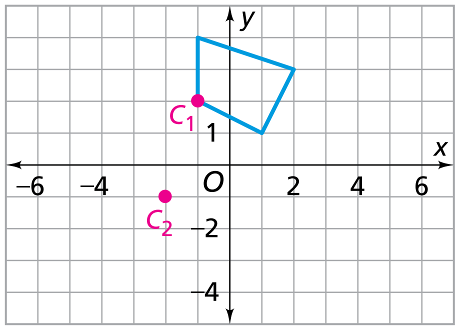
H. Reflecting the figure in the y-axis and then rotating the image 180° about the origin gives the same final image as reflecting the figure in the x-axis.

J. Rotating the figure 90° counterclockwise about the origin and then rotating the image another 90° counterclockwise gives the same image as rotating the original image 180° about the origin.

30. Investigate what happens when you rotate a figure 180° about a point and then rotate the image 180° about a different point. Is the combination of the two rotations equivalent to a single transformation? Test several cases and make a conjecture about the result.

You might start your investigation with the figures below. Copy them onto grid paper. Rotate each polygon 180° about point C_1 and then 180° about point C_2.





31. Plot points $P(-2, 4)$ and $Q(2, 1)$ on a coordinate grid.

a. Find the coordinates of the points P' and Q' that are the images of points P and Q after a reflection in the x-axis. Then, use the Pythagorean Theorem to prove that $PQ = P'Q'$.

b. Find coordinates of the points P'' and Q'' that are the images of points P and Q after a counterclockwise rotation of 180° about the origin. Then, prove that $PQ = P''Q''$.

c. Find coordinates of the points P''' and Q''' that are the images of points P and Q after a translation with the rule $(x, y) \rightarrow (x + 3, y - 5)$. Then, prove that $PQ = P'''Q'''$.