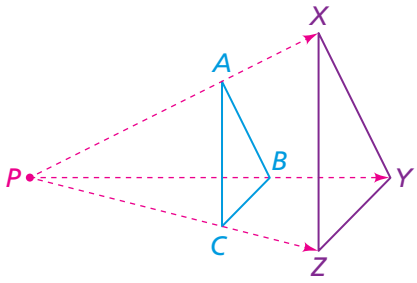
By measuring the figures that result from stretching or shrinking operations, you learned about **similarity transformations.** These are actions that change the size, but not the shape, of geometric figures. In this Investigation, you will review and extend your understanding of dilations and other similarity transformations.

4.1 Focus on Dilations

The key part of any similarity transformation is a **dilation.** The diagram below shows how a dilation, centered at point P with scale factor $\frac{3}{2}$ or 1.5, transforms triangle ABC to triangle XYZ. The two triangles are **similar figures.**



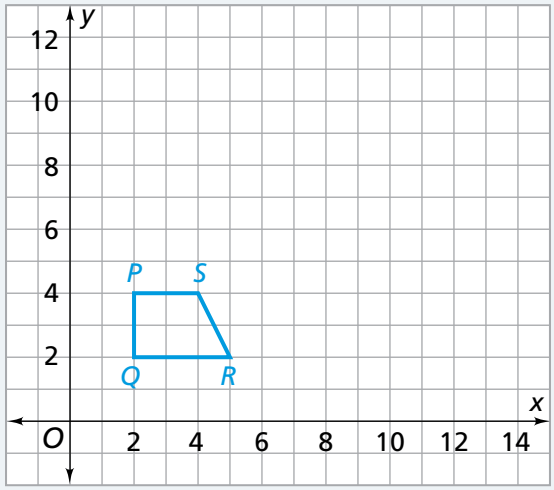
In everyday language, a "dilation" is usually an enlargement with a scale factor greater than 1. In mathematics, the scale factor of a dilation may be greater than or less than 1. A scale factor greater than 1 causes stretching, while a scale factor less than 1 causes shrinking. In fact, the diagram above also shows how a dilation with center P and scale factor $\frac{2}{3}$, or about 0.67, transforms triangle XYZ to triangle ABC.

How do dilations affect the size and shape of the figures they transform?

In this Problem, you will review the properties of stretching and shrinking transformations by working with figures on a coordinate grid.

Problem 4.1

A Copy the figure below onto grid paper. Draw the image of quadrilateral *PQRS* after a dilation with center (0, 0) and scale factor 3. Label corresponding points P', Q', R', and S'.



1. How do the side lengths of quadrilateral $P'Q'R'S'$ compare to those of quadrilateral *PQRS*?
2. How do the angle measures of quadrilateral $P'Q'R'S'$ compare to those of quadrilateral *PQRS*?
3. How does the perimeter of quadrilateral $P'Q'R'S'$ compare to that of quadrilateral *PQRS*?
4. How does the area of quadrilateral $P'Q'R'S'$ compare to that of quadrilateral *PQRS*?
5. How do the slopes of the sides of quadrilateral $P'Q'R'S'$ compare to the slopes of the sides of quadrilateral *PQRS*?
6. What rule of the form $(x, y) \rightarrow (\blacksquare, \blacksquare)$ shows how coordinates of corresponding points are related under a dilation with center (0, 0) and scale factor 3?

continued on the next page >

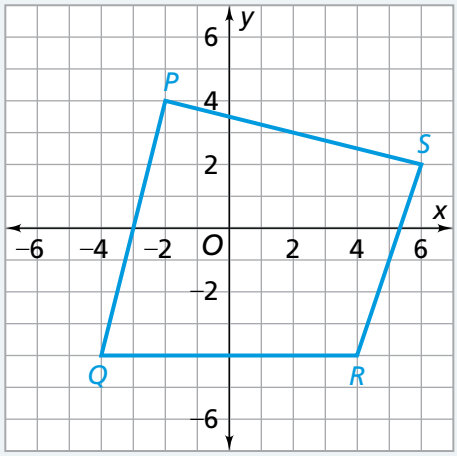
Problem 4.1 continued

B On the drawing from Question A, draw $\overline{OP}$, $\overline{OP'}$, $\overline{OQ}$, and $\overline{OQ'}$.

1. What similar triangles do you see? Explain.
2. **a.** Suppose point $Z = (2z, 4z)$. How are points P and Z related to each other?

   b. Use coordinates to find the slopes of $\overline{OP}$, $\overline{OP'}$, $\overline{OZ}$, and $\overline{PZ}$. What do you notice? Explain why your discovery makes sense.

C Copy the figure below onto grid paper. Draw the image of quadrilateral $PQRS$ after a dilation with center $(0, 0)$ and scale factor $\frac{1}{2}$.



1. How do the side lengths of quadrilateral $P'Q'R'S'$ compare to those of quadrilateral $PQRS$?
2. How do the angle measures of quadrilateral $P'Q'R'S'$ compare to those of quadrilateral $PQRS$?
3. How does the perimeter of quadrilateral $P'Q'R'S'$ compare to that of quadrilateral $PQRS$?
4. How does the area of quadrilateral $P'Q'R'S'$ compare to that of quadrilateral $PQRS$?
5. How do the slopes of the sides of quadrilateral $P'Q'R'S'$ compare to the slopes of the sides of quadrilateral $PQRS$?
6. What rule of the form $(x, y) \rightarrow (\blacksquare, \blacksquare)$ shows how coordinates of corresponding points are related under a dilation with center $(0, 0)$ and scale factor $\frac{1}{2}$?

Problem 4.1 continued

D Use your results from Questions A and B to write conjectures about the effects of a dilation with scale factor k on a polygon. If necessary, try dilating a few more figures.

Begin your conjectures with: "When two polygons are related by a dilation with scale factor k, . . . "

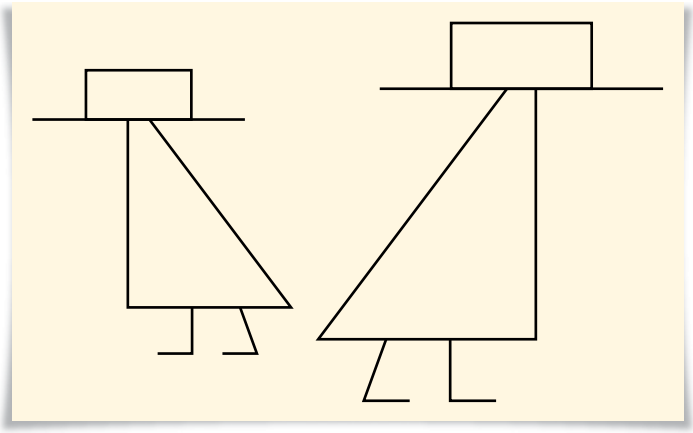
E If you dilate a figure with reflectional or rotational symmetry, will the resulting image have the same symmetry? Explain.

ACE Homework starts on page 86.

4.2 Return of Super Sleuth

Similarity Transformations

Dilations transform geometric figures to larger or smaller versions of the same shape. The diagram below shows two versions of the "Super Sleuth" logo for the P. I. Middle School Mystery Club. They appear to be similar, but there is no obvious center or scale factor for a dilation that would stretch or shrink one image onto the other.



? What strategies could you use to check the similarity of the two figures?