

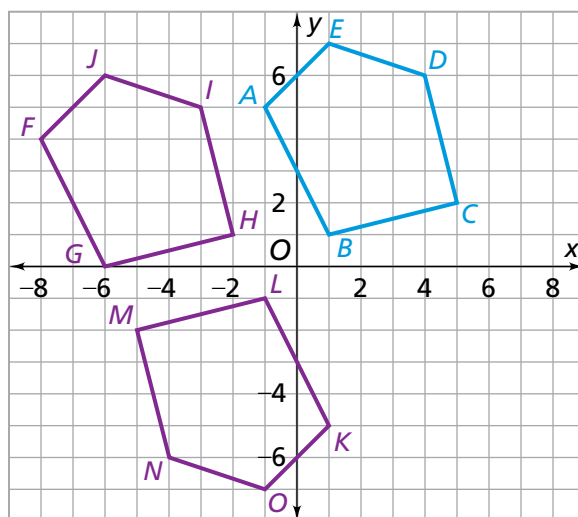
3.4 A Special Property of Translations and Half-Turns



When studying mathematical or scientific questions, asking yourself “What will happen if . . . ?” is helpful. In geometry, that means asking how properties of a figure will or will not change when you apply a transformation to it.

Your study of flips, turns, and slides showed that those transformations do not change the size or shape of a figure. Line segments “move” to line segments that are the same length. Angles “move” to angles of the same measure. In addition to these basic properties of transformations, translations and half-turn rotations have a special effect on lines.

The diagram below shows the effect of “moving” pentagon $ABCDE$ in two ways. The first is a translation to pentagon $FGHIJ$. The second is a half-turn or 180° rotation about the origin to pentagon $KLMNO$.



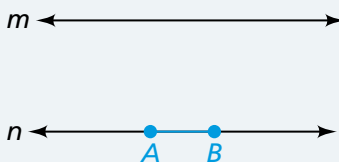
- What is the special relationship among the corresponding sides of the three figures?
- How can you use the coordinates of the vertices to prove your conjecture?

In the Problem, you will discover and prove the special relationship among corresponding sides of pentagon $ABCDE$ and its images after a translation and after a 180° rotation.

Problem 3.4



- A** Look at \overline{AB} and its image after a translation, \overline{FG} .
- In Investigation 1, you observed that a segment and its image after a translation appear to be congruent and parallel. Use the coordinates of the endpoints and slopes of lines to prove that your observation is correct.
 - Are other pairs of segments in pentagons $ABCDE$ and $FGHIJ$ related in the same way?
- B** Look at \overline{AB} and its image after a half-turn, \overline{KL} .
- How do the two segments appear to be related?
 - Use the coordinates of the endpoints to test your conjecture.
 - Are other pairs of segments in pentagons $ABCDE$ and $KLMNO$ related in the same way?
- C** Complete the following sentences to describe the pattern you found:
- A translation “moves” every line m to a line n so that . . .
 - A half-turn “moves” every line m to a line n so that . . .
- D** If lines m and n are parallel, will it always be possible to find a translation or half-turn that “moves” one line onto the other? If so, what point should you choose for the center of the rotation? Explain.



A C E Homework starts on page 61.