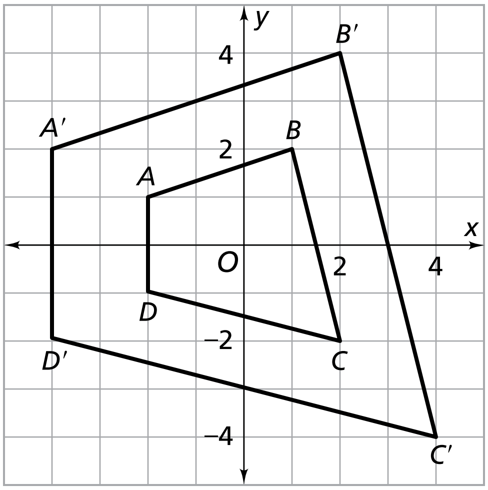
Applications

1. The figure and its image after dilation will look like this:



2. Side lengths of $ABCD$ are: $AB = \sqrt{10}$; $BC = \sqrt{17}$; $CD = \sqrt{17}$; and $DA = 2$

   Side lengths of $A'B'C'D'$ are: $A'B' = \sqrt{40}$; $B'C' = \sqrt{68}$; $C'D' = \sqrt{68}$; and $D'A' = 4$

   Side lengths of the dilated figure are double the length of the original.

3. The perimeter of $A'B'C'D'$ is about 26.8, which is double the perimeter of $ABCD$ which is about 13.4.

4. The area of $ABCD$ is 10.5 square units; the area of $A'B'C'D'$ is 42 square units, 4 times that of $ABCD$.

5. The slopes of the sides of $ABCD$ are: $\overline{AB}$ slope $\frac{1}{3}$; $\overline{BC}$ slope -4; $\overline{CD}$ slope $-\frac{1}{4}$; $\overline{DA}$ slope is undefined. The slopes of the corresponding sides of $A'B'C'D'$ are the same.

6. A dilation with scale factor $\frac{1}{2}$ centered at the origin will transform $A'B'C'D'$ exactly onto $ABCD$.

7. The side lengths of $A''B''C''D''$ will be:

   a. 2 times the corresponding side lengths of $ABCD$

   b. equal to the corresponding side lengths of $A'B'C'D'$

8. The perimeter of $A''B''C''D''$ will be:

   a. 2 times the perimeters of $ABCD$

   b. equal to the perimeter of $A'B'C'D'$

9. The area of $A''B''C''D''$ will be:

   a. 4 times the area of $ABCD$

   b. equal to the area of $A'B'C'D'$

10. The slopes of sides in $A''B''C''D''$ will be:

    a. equal to the slopes of corresponding sides in $ABCD$ if the second transformations are slides or 180° rotation, *but will not be equal for reflections or other rotations.*

    b. equal to the slopes of corresponding sides in $A'B'C'D'$ if the second transformations are slides or 180° rotations, *but will not be equal for reflections or other rotations.*

Note: For Exercises 11–15, student responses might vary depending on the accuracy of their angle and side measurement. The point should be that they have reasons for their conclusions.

11. These triangles appear to be similar, a fact that could be shown by rotating triangle PQR through an angle of 90° clockwise or 270° counterclockwise about R and then dilating centered at R with scale factor about $\frac{5}{3}$.

12. These triangles do not appear to be similar. The angles opposite the longest sides are not congruent, so the necessary correspondence of parts could not be shown.

13. These triangles appear to be similar, a fact that could be shown by reflecting triangle PQR across a line that is the perpendicular bisector of $\overline{RU}$ and then dilating centered at U with scale factor about $\frac{7}{5}$.

14. These rectangles appear to be similar since the ratios of length and width are close to the same. The similarity could be shown by rotating one of the rectangles through an angle of 90°, sliding it so that one of the corners is positioned on the corresponding corner in the other rectangle, and then dilating (or shrinking) with scale factor about 2.

15. These parallelograms do not appear to be similar. The ratios of corresponding sides are not the same in the two figures.

16. True: One could imagine "moving" triangle *ABC* on top of triangle *PQR* so the 24° angles match and then dilating from *P* with scale factor 2.5 to 'move' *ABC* exactly on top of *PQR*.

17. True: $\overline{AD}$ and $\overline{BE}$ are transversals cutting the parallel lines, so $\angle A \cong \angle D$ and $\angle B \cong \angle E$. This implies that the triangles are similar. It is also true that the vertical or opposite angles at point *C* are congruent.

18. True: All angles will measure 60° so the Angle-Angle-Angle criterion for similarity will be satisfied.

19. False: For example, all rectangles have all angles measuring 90°, but not all rectangles are similar.

20. False: The two triangles could have quite different vertex angles.

21. Measuring height of a tall building.

a. 96 feet

b. The triangles pictured are similar by the same reasoning applied in Problem 4.4 and the scale factor relating corresponding side lengths is $\frac{32}{2} = 16$.

22. Using shadows to form similar triangles.

a. The shorter building must be 57.6 feet tall.

b. The triangles are similar because at any specific time of day the suns rays strike the earth at essentially the same angle when one is looking in a small geographic region. The buildings are assumed to meet the ground at right angles, so the two triangles have two congruent corresponding angles.

The taller building is 96 feet high and the scale factor from larger to smaller is 0.6. We have 0.6(96) = 57.6.

Connections

23. Sphere of radius 5 cm.

a. Volume $= \left(\frac{4}{3}\right)\pi(5^3)$ or about 523.6 cm^3; Surface Area $= 4\pi(5^2)$ or about 314 cm^2

b. Scaling the sphere by a factor of 2:

i. Surface area becomes 4×314 or about 1,256 cm^2.

ii. Volume becomes 8×523.6 or about 4,189 cm^3.

c. Surface area always changes by the square of the scale factor; volume always changes by the cube of the scale factor.

24. Composition of dilations.

a. $(x, y) \rightarrow (6x, 6y)$

b. The rule would be the same if the composition occurs in the opposite order because multiplication is commutative.

25. Dilations and symmetry.

a. Yes, symmetry is preserved by dilation.

b. Scale factor change would not affect preservation of symmetry.

c. Using a different center of scaling would change the result. For example, if we use the point (5, 0) as the center then the entire figure will be to the left of the *y*-axis. The figure will still have reflectional symmetry, but the line of reflection will not be the *y*-axis.

26. True: All angles are 90°, and in each figure, the sides are the same length, so there is a single scale factor of dilation that would transform one square onto the other, after slides and turns to position one in a corner of the other.

27. False: The angles could be quite different in two rhombuses.

28. a. Similar triangles are *ABC*, *EDF*, *BAF*, *CDA*, and *EBC*. The scale factor from *ABD* to *EDF* is 2. The scale factor from *ABD* to *BAF*, *CDA*, and *EBC* is 1, which also makes *ABC* congruent to *BAF*, *CDA*, and *EBC*. Angles are "moved" to equal angles by the rotations named, so all these triangles have corresponding equal angles.

b. Parallelograms are *ABCD*, *ABEC*, and *AFBC*. The parallel sides are the result of half-turns.

Note: There is a theorem that states that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half its length. You can see this result in the figure for Exercise 28.

Extensions

29. Side-Side-Side similarity.

a. It turns out that they are right.

b. The reasoning given is correct. Dilations preserve angle measures; the fact that the dilated triangle has side lengths congruent to those in triangle *XYZ* follows from the given information about the two original triangles. Congruence of triangle $A'B'C'$ and triangle *XYZ* follows from the Side-Side-Side criterion for congruence; the congruence of corresponding angles is due to corresponding parts of congruent triangles. Since the angles of $A'B'C'$ are congruent to those of *ABC*, we have congruence of angles that is needed to show *ABC* similar to *XYZ*.

30. The composite of two dilations, even with different centers will dilate with a scale factor that is the product of the two component scale factors. To find the center of the composite dilation, one only needs to locate two final image points and draw rays from those points through the points from which they "started" in motion. The intersection point of those two rays will be the center of the composite dilation.

31. Dilation in one direction.

a. The one-directional dilation does not produce similar image figures.

b. There is no simple rule for predicting the effect of these one-directional dilations on side lengths, perimeters, or angle measures because the position of the original figure matters. For the rule given, vertical sides will not change in length but horizontal sides will be doubled in length. Some angles will get larger [e.g. the angle determined by (1, 1), (0, 0), and (−1, 1)], some will get smaller [e.g., the angle determined by (1,1), (0, 0), and (1, −1)], and some will stay the same measure (e.g, the intersection of the *x*- and *y*-axes).

The somewhat surprising result is for any polygon (or even circle or irregular figure). The area will be multiplied by 2. If you imagine a figure covered by unit squares, the one-directional dilation stretches each into a 1×2 rectangle with area 2 square units.

c. The slope of any line will be cut in half by this transformation. $\frac{y_1 - y_2}{2x_1 - 2x_2} = \frac{1}{2}\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$.