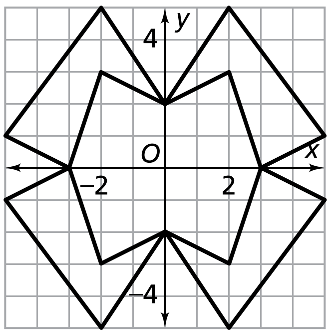
Applications

1. Coordinates of key points and their images are shown in Figure 1.

   a. For Exercises 1–3, the figure and its image after reflection in the *y*-axis (top right), the *x*-axis (bottom left), and both axes (bottom right) will look like this:



   b. $(x, y) \rightarrow (-x, y)$

2. Coordinates of key points and their images are shown in Figure 2.

   a. See diagram in Exercise 1, bottom left figure.

   b. $(x, y) \rightarrow (x, -y)$

3. Coordinates of key points and their images are shown in Figure 3.

   a. See diagram in Exercise 1, bottom right figure.

   b. $(x, y) \rightarrow (-x, -y)$

   c. The composite of successive line reflections in perpendicular lines is always equal to a half-turn.

Figure 1

Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(–5, 1)	(–2, 5)	(0, 2)	(–2, 3)	(–3, 0)
Coordinates After a Reflection in *y*-axis	(5, 1)	(2, 5)	(0, 2)	(2, 3)	(3, 0)

Figure 2

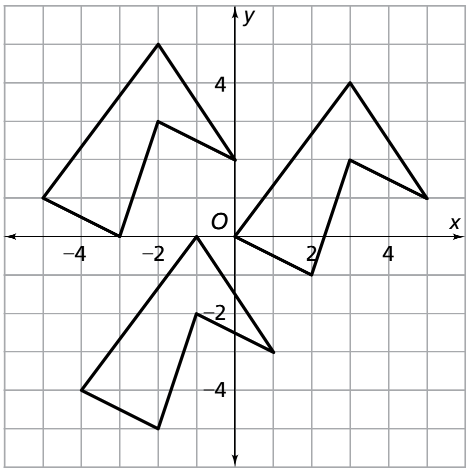
Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(–5, 1)	(–2, 5)	(0, 2)	(–2, 3)	(–3, 0)
Coordinates After a Reflection in *x*-axis	(–5, –1)	(–2, –5)	(0, –2)	(–2, –3)	(–3, 0)

Figure 3

Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(–5, 1)	(–2, 5)	(0, 2)	(–2, 3)	(–3, 0)
Coordinates After a Reflection in *x*-axis	(5, –1)	(2, –5)	(0, –2)	(2, –3)	(3, 0)

4. A table that shows coordinates of key points on the figure and their images after the translation that "moves" point B to the point with coordinates (3, 4) will look like Figure 4.

a. For Exercises 4 and 5, the figure and its images after the two translations will look like this (right figure first and then bottom figure second):



b. $(x, y) \rightarrow (x + 5, y - 1)$.

5. The table will look like Figure 5 for the second translation.

a. See bottom figure in Exercise 4.

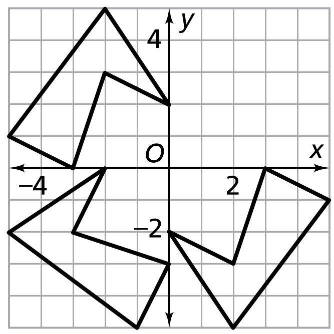
b. $(x, y) \rightarrow (x - 4, y - 4)$.

c. $(x, y) \rightarrow (x + 1, y - 5)$.

d. The composite is a translation that moves every original point 1 unit to the right and down 5 units.

6. The table will look like Figure 6.

a. For Exercises 6 and 7, the original image (top left), the image after one rotation of 90° (bottom left), and the image after two rotations of 90° (bottom right) will look like this:



b. $(x, y) \rightarrow (-y, x)$.

Figure 4

Point	A	B	C	D	E
Original Coordinates	(−5, 1)	(−2, 5)	(0, 2)	(−2, 3)	(−3, 0)
Coordinates After Translating B to (3, 4)	(0, 0)	(3, 4)	(5, 1)	(3, 2)	(2, −1)

Figure 5

Point	A	B	C	D	E
Original Coordinates	(−5, 1)	(−2, 5)	(0, 2)	(−2, 3)	(−3, 0)
Coordinates after translating B to (3, 4)	(0, 0)	(3, 4)	(5, 1)	(3, 2)	(2, −1)
Coordinates After Translating B' to (−1, 0)	(−4, −4)	(−1, 0)	(1, −3)	(−1, −2)	(−2, −5)

Figure 6

Point	A	B	C	D	E
Original Coordinates	(−5, 1)	(−2, 5)	(0, 2)	(−2, 3)	(−3, 0)
Coordinates After a 90° Rotation	(−1, −5)	(−5, −2)	(−2, 0)	(−3, −2)	(0, −3)

7. The table of coordinates for key points will look like Figure 7.

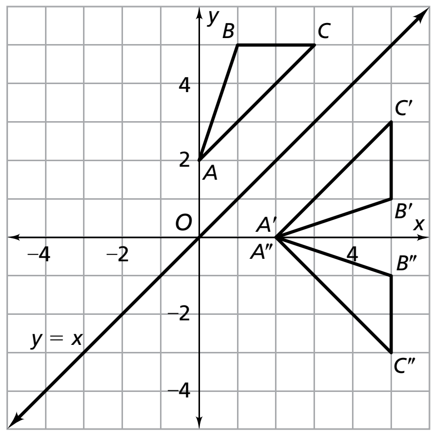
b. $(x, y) \rightarrow (-x, -y)$.

c. The composite is a half-turn.

8. Composites of line reflections.

a. The table (after both reflections) will look like Figure 8.

b–c. The image of the triangle after reflection in $y = x$ is the top right figure; the image after a subsequent reflection in the x-axis is the bottom right figure.



d–e. The image after first reflecting in the x-axis is the bottom right figure and then after reflecting in the line $y = x$ is the left figure. Comparing this drawing to that in parts (a)–(c) shows that composition of line reflections is not commutative. It is a general property of such compositions that the result is a rotation about the point of intersection of the lines through an angle that is double the angle between the two lines (from first line to second line). **Note:** That is more than we expect students to get out of this Exercise. It is revisited in Extensions.

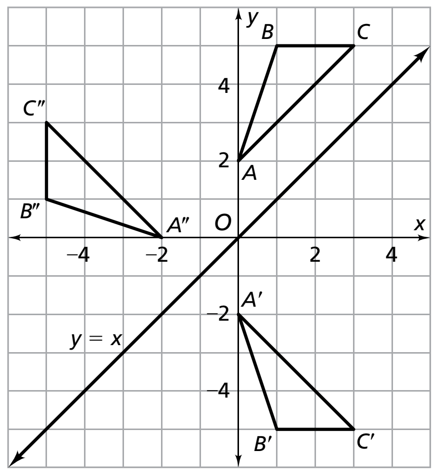


Figure 7

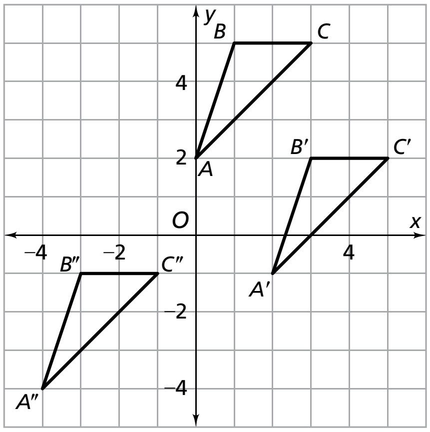
Point	*A*	*B*	*C*	*D*	*E*
Original Coordinates	(–5, 1)	(–2, 5)	(0, 2)	(–2, 3)	(–3, 0)
Coordinates After a 90° Rotation	(–1, –5)	(–5, –2)	(–2, 0)	(–3, –2)	(0, –3)
Coordinates After a 180° Rotation	(5, –1)	(2, –5)	(0, –2)	(2, –3)	(3, 0)

Figure 8

Point	*A*	*B*	*C*
Original Coordinates	(0, 2)	(1, 5)	(3, 5)
Coordinates After a Reflection in $y = x$	(2, 0)	(5, 1)	(5, 3)
Coordinates After a Reflection in x-axis	(2, 0)	(5, –1)	(5, –3)

9. Translation effects.

a. The image after translation (i) is the right-most figure; the image after translation (ii) is the bottom left figure.



b. Comparing slopes

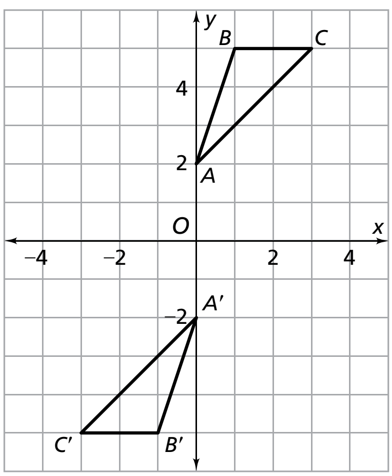
i. The slopes of $\overline{AB}$ and $\overline{A'B'}$ are both 3; the slopes of $\overline{AC}$ and $\overline{A'C'}$ are both 1; the slopes of $\overline{CB}$ and $\overline{C'B'}$ are both 0.

ii. The slopes of $\overline{AB}$ and $\overline{A''B''}$ are both 3; the slopes of $\overline{AC}$ and $\overline{A''C''}$ are both 1; the slopes of $\overline{CB}$ and $\overline{C''B''}$ are both 0.

c. Translations preserve slopes of lines and map lines onto parallel lines. This will be confirmed in Problem 3.4.

10. Half-turn effects.

a. The image triangle is the bottom figure.



b. Comparing slopes.

i. Slopes of $\overline{AB}$ and $\overline{A'B'}$ are both 3.

ii. Slopes of $\overline{AC}$ and $\overline{A'C'}$ are both 1.

iii. Slopes of $\overline{CB}$ and $\overline{C'B'}$ are both 0.

c. The results support the discovery in work on Problem 3.4 that half-turns preserve slopes of lines and map lines onto parallel lines.

11. Angles *a*, *c*, and *e* all measure 120°. Angles *b*, *d*, *f*, and *g* all measure 60°.

12. Angle *a* measures 135°. Angle *b* measures 15°.

13. $x = 30$, $2x = 60$, and $3x = 90$.

14. Parallelogram properties

a. Assuming only that opposite sides are parallel (and $\overline{DB}$ is a transversal to both pairs of parallel sides), we can conclude that $\angle ADB \cong \angle CBD$ and $\angle ABD \cong \angle CDB$ because they are the alternate interior angles formed with parallel lines cut by a transversal.

b. The two triangles are congruent because they share a common side *DB* and thus satisfy the two angles with an included side, i.e., Angle-Side-Angle congruence criterion.

c. The opposite angles are corresponding parts of congruent triangles.

d. The opposite sides are also corresponding parts of congruent triangles.

15. Diagonals of a rectangle.

a. Any rectangle is a parallelogram, because opposite sides are perpendicular to a common side and thus parallel to each other.

So opposite sides $\overline{AD}$ and $\overline{BC}$ are congruent (from 14). Also, side $\overline{AB}$ is common to the two triangles, and $\angle DAB \cong \angle CBA$ because both are right angles. This gives criteria for triangle congruence by two sides with an included angle, i.e., the Side-Angle-Side result.

One could also make an argument using two angles with an included angle, i.e., the Angle-Side-Angle result, which mimics the work in Exercise 14.

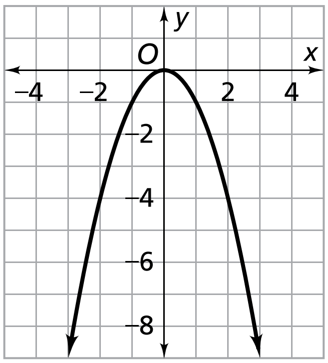
b. Congruence of the diagonals follows from congruence of the triangles in which they are corresponding parts.

Connections

16. The sample table of values will look like this:

x	−3	−2	−1	0	1	2	3
$y = -x^2$	−9	−4	−1	0	−1	−4	−9

a. The graph (with points between data from table filled in) will look like this:

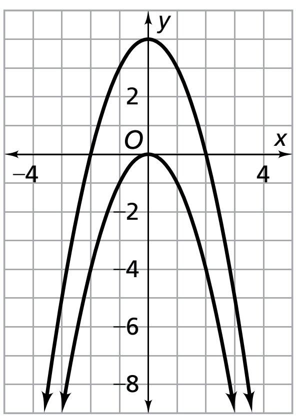


b. This graph is also symmetric about the y-axis.

17. The sample table of values will now look like this:

x	−3	−2	−1	0	1	2	3
$y = -x^2$	−9	−4	−1	0	−1	−4	−9
$y = -x^2 + 4$	−5	0	3	4	3	0	−5

a. The addition of the new function will yield a graph like this:



b. Transformations that match graphs.

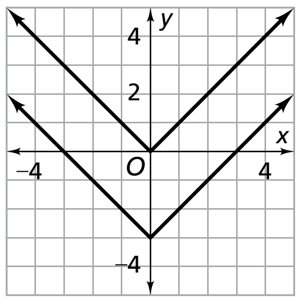
i. Translation up 4 units with rule: $(x, y) \rightarrow (x, y + 4)$

ii. Translation down of 4 units with rule: $(x, y) \rightarrow (x, y - 4)$

18. The absolute value function.

x	−4	−3	−2	−1	0	1	2	3	4
$y = \|x\|$	4	3	2	1	0	1	2	3	4

a. Graphs of the absolute value function and the variation called for in Exercise 19 will look like this:



b. The graph is symmetric about the y-axis.

19. The table including $y = |x| - 3$ will look like this:

x	−4	−3	−2	−1	0	1	2	3	4
$y = \|x\|$	4	3	2	1	0	1	2	3	4
$y = \|x\| - 3$	1	0	−1	−2	−3	−2	−1	0	1

a. See graph in answer to Exercise 18, part (a).

b. Transformations to match graphs.

i. A translation with rule $(x, y) \rightarrow (x, y - 3)$ will slide the original graph down.

ii. A translation with rule $(x, y) \rightarrow (x, y + 3)$ will slide the original graph up.

20. **a.** $3 > -1$

b. $8 > 4$

c. $-2 > -6$

d. $-3 < 1$

e. If a and b are numbers on a number line and $a < b$, then $a + c < b + c$. (Like translating on the x-axis.)

f. If a and b are numbers on a number line and $a < b$, then $a(-1) > b(-1)$. (Like rotating the x-axis about 0.)

21. The given design appears to have reflectional symmetry across vertical and horizontal axes and half-turn symmetry.

22. The given design appears to have six lines of reflectional symmetry through vertices and midpoints of sides of the interior hexagon and rotational symmetries in multiples of 60°.

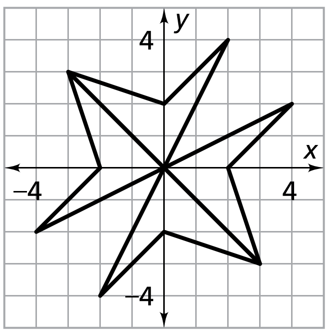
23. B; the only correct statement about all parallelograms.

24. The area of the triangle is 3 square units.

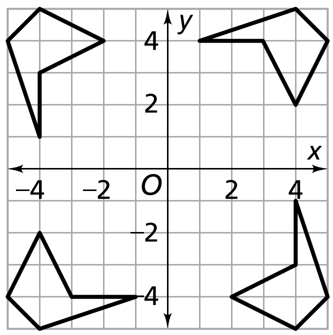
25. The side lengths are 2, $\sqrt{10}$, and $\sqrt{18}$; the perimeter is the sum of these numbers which comes to approximately 9.4.

Extensions

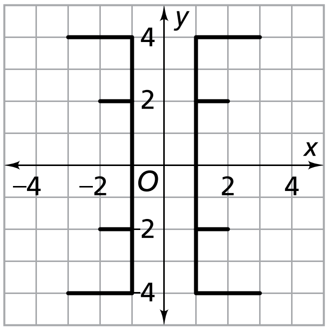
26. There are a number of ways that this can be done. Here's one design that was constructed by reflecting the given basic design element across the line $y = x$ and then the resulting two-part figure over the line $y = -x$.



27. The simplest way to make a rotation-symmetric figure with the given basic design element is to use a half-turn of the given figure to create a two-part design. The following sketch shows what happens if 90° rotations are used (centered about the origin).



28. Again, there are probably many ways to satisfy the requirements for both reflectional and rotational symmetry. The following drawing shows the result of reflecting the given letter F across the *y*-axis and then reflecting the resulting two-part figure across the *x*-axis. The four-part final figure has two lines of reflectional symmetry and also half-turn rotational symmetry.



29. G; that is, this is the incorrect statement.

30. The composite of two half-turns about different centers is a translation in the direction and twice the distance of a line segment from the first center to the second center.

31. Each segment of interest can be seen to be the hypotenuse of a right triangle with legs 3 and 4, so $PQ = 5$.

a. P' has coordinates $(-2, -4)$ and Q' has coordinates $(2, -1)$, so $P'Q' = 5$.

b. P'' has coordinates $(2, -4)$ and Q'' has coordinates $(-2, -1)$, so $P''Q'' = 5$.

c. P''' has coordinates $(1, -1)$ and Q''' has coordinates $(5, -4)$, so $P'''Q''' = 5$.