

Systems of Equations Word Problems

1) Find the value of two numbers if their sum is 12 and their difference is 4.

Let $x = 1^{st}$ number
Let $y = 2^{nd}$ number

$$\begin{cases} x+y=12 \\ x-y=4 \end{cases}$$

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$$\underline{\quad\quad}$$

$$2x=16$$

$$\frac{2x}{2} = \frac{16}{2}$$

$$x=8$$

$$\begin{aligned} x+y &= 12 \\ (-8)+y &= 12 \\ -8 & \quad -8 \\ \hline y &= 4 \end{aligned}$$

1^{st} number = 8
 2^{nd} number = 4

2) The difference of two numbers is 3. Their sum is 13. Find the numbers.

Let $x = 1^{st}$ number
Let $y = 2^{nd}$ number

$$\begin{cases} x-y=3 \\ x+y=13 \end{cases}$$

$$\begin{cases} x-y=3 \\ x+y=13 \end{cases}$$

$$\underline{\quad\quad}$$

$$2x=16$$

$$\frac{2x}{2} = \frac{16}{2}$$

$$x=8$$

$$\begin{aligned} x-y &= 3 \\ 8-y &= 3 \\ -8 & \quad -8 \\ \hline -y &= -5 \\ -1 & \quad -1 \\ \hline y &= 5 \end{aligned}$$

1^{st} number = 8
 2^{nd} number = 5

3) Flying to Kampala with a tailwind a plane averaged 158 km/h. On the return trip the plane only averaged 112 km/h while flying back into the same wind. Find the speed of the wind and the speed of the plane in still air.

Let $x = \text{speed of the plane}$
Let $y = \text{speed of the wind}$

$$\begin{cases} x+y=158 \\ x-y=112 \end{cases}$$

$$\begin{cases} x+y=158 \\ x-y=112 \end{cases}$$

$$\underline{\quad\quad}$$

$$2x=270$$

$$\frac{2x}{2} = \frac{270}{2}$$

$$x=135$$

$$\begin{aligned} x+y &= 158 \\ 135+y &= 158 \\ -135 & \quad -135 \\ \hline y &= 23 \end{aligned}$$

Plane Speed = 135 km/hr
Wind Speed = 23 km/hr

4) The school that Stefan goes to is selling tickets to a choral performance. On the first day of ticket sales the school sold 3 senior citizen tickets and 1 child ticket for a total of \$38. The school took in \$52 on the second day by selling 3 senior citizen tickets and 2 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

Let $x = \text{price of senior ticket}$
Let $y = \text{price of child ticket}$

$$\begin{cases} 3x+2y=52 \\ 3x+y=38 \end{cases}$$

$$\underline{\quad\quad}$$

$$y=14$$

$$\begin{aligned} 3x+y &= 38 \\ 3x+14 &= 38 \\ -14 & \quad -14 \\ \hline 3x &= 24 \\ \frac{3x}{3} & \quad \frac{24}{3} \\ x &= 8 \end{aligned}$$

Senior Ticket costs \$8
Child Ticket costs \$14

5) The sum of the digits of a certain two-digit number is 7. Reversing its digits increases the number by 9. What is the number?

Let $x = \text{one digit}$
Let $y = \text{other digit}$

$$\begin{aligned} 10x+y &= 10y+x-9 \\ -10y & \quad -10y \\ \hline 10x-9y &= x-9 \\ -x & \quad -x \\ \hline 9x-9y &= -9 \end{aligned}$$

$$\begin{cases} x+y=7 \\ 9x-9y=-9 \end{cases} \Rightarrow \begin{aligned} 9x+9y &= 63 \\ 9x-9y &= -9 \\ \hline 18x &= 54 \\ \frac{18x}{18} & \quad \frac{54}{18} \\ x &= 3 \end{aligned}$$

$$\begin{aligned} x+y &= 7 \\ 3+y &= 7 \\ -3 & \quad -3 \\ \hline y &= 4 \end{aligned}$$

The number is 34

6) A boat traveled 210 miles downstream and back. The trip downstream took 10 hours. The trip back took 70 hours. What is the speed of the boat in still water? What is the speed of the current?

Let $x = \text{speed of the boat}$
Let $y = \text{speed of the current}$

$$\begin{cases} 10x+10y=210 \\ 70x-70y=210 \end{cases}$$

$$\begin{aligned} 70x+70y &= 1470 \\ 70x-70y &= 210 \\ \hline 140x &= 1680 \\ \frac{140x}{140} & \quad \frac{1680}{140} \\ x &= 12 \end{aligned}$$

$$\begin{aligned} 10x+10y &= 210 \\ 10(12)+10y &= 210 \\ 120+10y &= 210 \\ -120 & \quad -120 \\ \hline 10y &= 90 \\ \frac{10y}{10} & \quad \frac{90}{10} \\ y &= 9 \end{aligned}$$

$$\begin{cases} 10x+10y=210 \\ 70x-70y=210 \end{cases}$$

Boat: 12 mph
Current: 9 mph

- 7) The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Let x = # of students/van
Let y = # of students/bus

$$\begin{cases} 8x + 8y = 240 \\ 4x + y = 54 \end{cases}$$

$$\begin{cases} 8x + 8y = 240 \\ 4x + y = 54 \end{cases} \Rightarrow \begin{cases} 8x + 8y = 240 \\ -8x - 2y = -108 \end{cases}$$

$$\begin{array}{r} 6y = 132 \\ \div 6 \\ y = 22 \end{array}$$

$$\begin{array}{r} 4x + y = 54 \\ 4x + 22 = 54 \\ -22 \quad -22 \\ \hline 4x = 32 \\ \div 4 \\ x = 8 \end{array}$$

8 students/van
22 students/bus

- 8) The senior classes at High School A and High School B planned separate trips to New York City. The senior class at High School A rented and filled 1 van and 6 buses with 372 students. High School B rented and filled 4 vans and 12 buses with 780 students. Each van and each bus carried the same number of students. How many students can a van carry? How many students can a bus carry?

Let x = # of students/van
Let y = # of students/bus

$$\begin{cases} x + 6y = 372 \\ 4x + 12y = 780 \end{cases}$$

$$\begin{cases} x + 6y = 372 \\ 4x + 12y = 780 \end{cases}$$

$$\begin{array}{r} 4x + 24y = 1488 \\ 4x + 12y = 780 \\ \hline 12y = 708 \\ \div 12 \\ y = 59 \end{array}$$

$$\begin{array}{r} x + 6y = 372 \\ x + 6(59) = 372 \\ x + 354 = 372 \\ -354 \quad -354 \\ \hline x = 18 \end{array}$$

18 Students/Van
59 Students/Bus

- 9) Brenda's school is selling tickets to a spring musical. On the first day of ticket sales the school sold 3 senior citizen tickets and 9 child tickets for a total of \$75. The school took in \$67 on the second day by selling 8 senior citizen tickets and 5 child tickets. What is the price each of one senior citizen ticket and one child ticket?

Let x = price of senior ticket
Let y = price of child ticket

$$\begin{cases} 3x + 9y = 75 \\ 8x + 5y = 67 \end{cases}$$

$$\begin{cases} 3x + 9y = 75 \\ 8x + 5y = 67 \end{cases}$$

$$\begin{array}{r} 24x + 72y = 600 \\ 24x + 15y = 201 \\ \hline 57y = 399 \\ \div 57 \\ y = 7 \end{array}$$

$$\begin{array}{r} 3x + 9y = 75 \\ 3x + 9(7) = 75 \\ 3x + 63 = 75 \\ -63 \quad -63 \\ \hline 3x = 12 \\ \div 3 \\ x = 4 \end{array}$$

Senior Ticket = \$4
Student Ticket = \$7

- 10) Matt and Ming are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Matt sold 3 small boxes of oranges and 14 large boxes of oranges for a total of \$203. Ming sold 11 small boxes of oranges and 11 large boxes of oranges for a total of \$220. Find the cost each of one small box of oranges and one large box of oranges.

Let x = cost of a small box
Let y = cost of a large box

$$\begin{cases} 3x + 14y = 203 \\ 11x + 11y = 220 \end{cases}$$

$$\begin{cases} 33x + 154y = 2233 \\ 33x + 99y = 660 \end{cases}$$

$$\begin{array}{r} 121y = 1573 \\ \div 121 \\ y = 13 \end{array}$$

$$\begin{array}{r} 3x + 14(13) = 203 \\ 3x + 182 = 203 \\ -182 \quad -182 \\ \hline 3x = 21 \\ \div 3 \\ x = 7 \end{array}$$

Small Box = \$7
Large Box = \$13

- 11) A boat traveled 336 miles downstream and back. The trip downstream took 12 hours. The trip back took 14 hours. What is the speed of the boat in still water? What is the speed of the current?

Let x = speed of the boat
Let y = speed of the current

$$\begin{cases} 12x + 12y = 336 \\ 14x - 14y = 336 \end{cases}$$

$$\begin{cases} 12x + 12y = 336 \\ 14x - 14y = 336 \end{cases}$$

$$\begin{array}{r} 2x + 2y = 56 \\ 2x - 2y = 48 \\ \hline 4x = 104 \\ \div 4 \\ x = 26 \end{array}$$

$$\begin{array}{r} 12x + 12y = 336 \\ 12(26) + 12y = 336 \\ 312 + 12y = 336 \\ -312 \quad -312 \\ \hline 12y = 24 \\ \div 12 \\ y = 2 \end{array}$$

Boat Speed = 26 mph
Current Speed = 2 mph