Applications

1. A school is planning a Saturday Back-to-School Festival to raise funds for the school art and music programs. Some of the planned activities are a ring toss, frog jump, basketball free throws, and a golf putting green. The organizers are considering two pricing plans.

Plan 1: \$5 admission fee, \$1 per game

Plan 2: \$2.50 admission fee, \$1.50 per game

a. Write equations that show how the cost y for playing the games at the festival depends on the number of games x that a participant chooses to play.

b. Estimate the coordinates of the intersection point of the graphs of the two equations. Check to see if those coordinates are an exact solution of both equations.

c. Use the expressions in the two cost equations to write and solve a single linear equation for the x-coordinate of the intersection point. Then use that x-value to find the y-coordinate of the intersection point.

d. For what number of games would Plan 1 be a better deal for participants than Plan 2?

2. In Exercise 1, suppose the two pricing plans changed as follows. Complete parts (a)–(d) based on these two plans.

Plan 1: \$4.50 admission fee, \$1 per game

Plan 2: \$3.50 admission fee, \$1 per game

Solve each system of equations.

3. $\begin{cases} y = 6x + 4 \\ y = 4x - 2 \end{cases}$

4. $\begin{cases} y = 3x + 7 \\ y = 5x - 7 \end{cases}$

5. $\begin{cases} y = -2x - 9 \\ y = 12x + 19 \end{cases}$

6. $\begin{cases} y = -x + 16 \\ y = -x - 8 \end{cases}$

7. $\begin{cases} y = 17x - 6 \\ y = 12x + 44 \end{cases}$

8. $\begin{cases} y = -20x + 14 \\ y = -8x - 44 \end{cases}$

For Exercises 9–14, write the equation in $y = mx + b$ form.

9. $4x + 6y + 12 = 0$

10. $-7x + 9y + 4 = 0$

11. $-4x - 2y - 6 = 0$

12. $-x + 4y = 0$

13. $2x - 2y + 2 = 0$

14. $25x + 5y - 15 = 0$

15. A sixth-grade class sells pennants and flags. They earn \$1 profit for each pennant sold and \$6 profit for each flag sold. They sell 50 items in total for a profit of \$115.

a. Write two equations that represent the relationship between the number of pennants sold p and the number of flags sold f.

b. How many pennants and how many flags were sold?

16. A seventh-grade class sells mouse pads and cell phone cases with their school logo on them. The class earns \$2 profit for each mouse pad sold and \$4 profit for each cell phone case sold. They sell 100 items in total for a profit of \$268.

a. Write two equations that represent the relationship between the number of mouse pads sold m and the number of cell phone cases sold c.

b. How many mouse pads and how many cell phone cases were sold?

17. Write a system of equations that you can use to find the two numbers.

a. Two numbers have a sum of 119 and a difference of 25.

b. Two numbers have a sum of 71 and a difference of 37.

c. Two numbers have a sum of 32 and a difference of 60.

d. Two numbers have a sum of 180 and a difference of 45.

e. If you know the sum and difference of two numbers, how can you use this information to find one of the two missing numbers? How do you find the second missing number?

18. On a hot summer day, Jay set up a lemonade stand. He kept track of how many glasses he sold on his phone.

a. Write two equations that relate the number of large glasses sold l and the number of small glasses sold s.

b. Solve the system of equations.

c. How many small glasses were sold?

d. How many large glasses were sold?

Pablo and Jasmine decide to try some other food trucks after eating at the taco truck in Problem 2.2. For Exercises 19–22, do the following.

a. Write two equations based on the information.

b. Solve the system of equations to determine the price of 1 serving of food and the price of 1 drink or bag of chips.

19. Pablo buys 3 servings of jambalaya and 2 drinks for $18.00.
Jasmine buys 1 serving of jambalaya and 2 drinks for $9.00.

20. Pablo buys 4 sandwiches and 4 bags of chips for $24.00.
Jasmine buys 8 sandwiches and 4 bags of chips for $43.00.

21. Pablo buys 3 loaves of zucchini bread and 5 cups of tea for $15.00.
Jasmine buys 5 loaves of zucchini bread and 3 cups of tea for $21.00.

22. Pablo buys 6 apple pies and 2 juices for $39.00.
Jasmine buys 2 apple pies and 4 juices for $18.00.

Solve each system by using the combination method.

23. $\begin{cases} 3x - 2y = 12 \\ -3x + 8y = -6 \end{cases}$

24. $\begin{cases} 4x + 9y = 7 \\ 4x - 9y = 9 \end{cases}$

25. $\begin{cases} 12x - 14y = -8 \\ -8x - 14y = 52 \end{cases}$

26. $\begin{cases} 5x + 15y = 10 \\ 5x - 10y = -40 \end{cases}$

27. $\begin{cases} -6x - 4y = 21 \\ -6x + 3y = 0 \end{cases}$

28. $\begin{cases} 2x - 3y = 14 \\ -x + 3y = -6 \end{cases}$

29. $\begin{cases} 3x + 2y = 17 \\ -2x - y = -12 \end{cases}$

30. $\begin{cases} 4x + 3y = 18 \\ 3x + 4y = 3 \end{cases}$

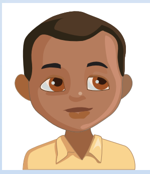
31. $\begin{cases} -2x + 6y = 42 \\ 4x - 3y = -12 \end{cases}$

32. Students in Mr. Coutley's class are playing the game "guess the date." For example, one student chooses a date (April 16), writes the date as an ordered pair (4, 16), and gives two clues.

Clue 1 "If I add the month number and the day number, the sum is 20."

Clue 2 "If I double the month number and add it to the day number, the sum is 24."

The other students try to determine the date based on the two clues. Find the date that each student is thinking of by writing and solving a system of two equations.



ANDREW

First Clue: If I double the month number and add it to double the day number, the sum is 26.

Second Clue: If I double the month number and then subtract double the day number, the difference is –18.

LARA

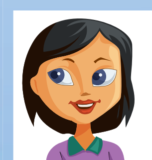
First Clue: If I double the month number and add the day number, the sum is 26.

Second Clue: If I multiply the month number by 10 and then subtract the day number, the difference is 10.

MARTHA

First Clue: The sum of triple the month number and double the day number is 62.

Second Clue: The difference of double the month number and the day number is 4.



AHNA

First Clue: The sum of four times the month number and the day number is 42.

Second Clue: The sum of the month number and four times the day number is 33.

Connections

For Exercises 33–38, solve the equation. Check the solution.

33. $3x + 12 = 24$

34. $-7x - 13 = 15$

35. $8 - 2x = 30$

36. $-7 + 9x = 38$

37. $-4 - 6x = -22$

38. $8x + 17 = -15$

39. For each part (a)–(f), find the value of y when $x = -2$.

a. $y = 3x - 7$

b. $3x - 2y = 10$

c. $7x - 4y = 12$

d. $x = 4y - 2$

e. $3 = 2x - y$

f. $12 = -3x - 4y$

Write an equation of the line satisfying the given conditions.

40. slope $= -4$, y-intercept $= 3$

41. slope $= \frac{2}{3}$, passes through the point (3, 4)

42. slope $= -3$, y-intercept $= 2$

43. passes through the points (5, 4) and (1, 7)

For Exercises 44–49, identify the slope and *y*-intercept of the line.

44. $3x + 2y = 4$

45. $4x - 8y = 12$

46. $x - y = 7$

47. $y = 4x - 8$

48. $2y = 4x + 6$

49. $y = 9$

50. Two lines can intersect at 0 points (if they are parallel), 1 point, or an infinite number of points (if they are the same). In parts (a)–(d), give all the possible numbers of intersection points for the two figures. Make sketches to illustrate the possibilities.

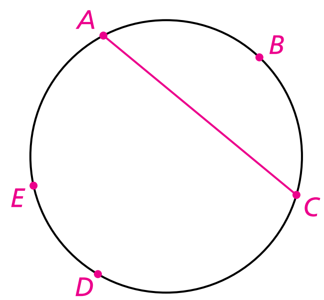
a. a circle and a straight line

b. two circles

c. a circle and a triangle

d. a circle and a rectangle

51. A **chord** is a line segment joining two points on a **circle.** $\overline{AC}$ is a chord in the diagram at the right.



a. How many chords can you draw by joining the labeled points on this circle?

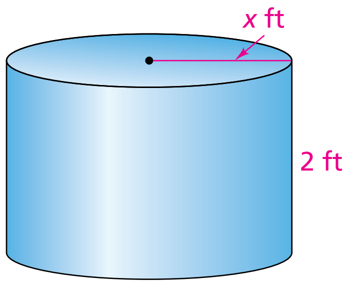
b. How many points inside the circle are intersection points of two or more of the chords from part (a)?

c. The chords cut the circle into several nonoverlapping regions. How many regions are formed?

52. **Multiple Choice** Which point is *not* on the graph of $2x - 5y = 13$?

A. $(9, 1)$ **B.** $(4, -1)$ **C.** $(0, 3.2)$ **D.** $(6.5, 0)$

53. The cylinder below represents an air conditioner with a radius of x feet and height of 2 feet.



a. Draw a net of a cover for the air conditioner. The top and sides need to be covered, but not the bottom.

b. Which equation below represents the area of the cover? Which represents the volume?

$y = 2\pi x^2$ $\qquad$ $y = \pi x^2 + 4\pi x$

$y = 2x^3$ $\qquad$ $y = \pi x(x + 4)$

54. **Multiple Choice** Kaya wants to fence off part of her yard for a garden. She has 150 feet of fencing. She wants a rectangular garden with a length 1.5 times its width. Which system represents these conditions?

F. $\begin{cases} 1.5w = \ell \\ w + \ell = 150 \end{cases}$ **G.** $\begin{cases} w = 1.5\ell \\ w + \ell = 150 \end{cases}$

H. $\begin{cases} 2w = 3\ell \\ w + \ell = 75 \end{cases}$ **J.** $\begin{cases} 3w = 2\ell \\ 2(w + \ell) = 150 \end{cases}$

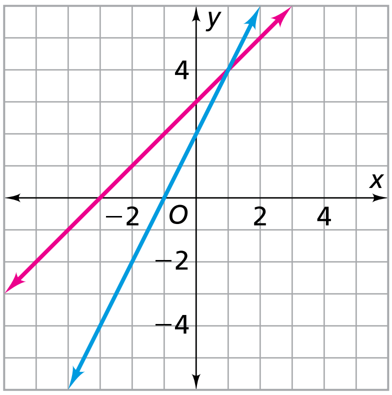
55. Multiple Choice Which equation shows how to find one dimension of the garden described in Exercise 54?

A. $2.5w = 150$ **B.** $2.5\ell = 150$

C. $2w = 3(75 - w)$ **D.** $5w = 150$

For Exercises 56–59, write an equation that represents each line on the graph. Then, solve the system of equations symbolically.

56.



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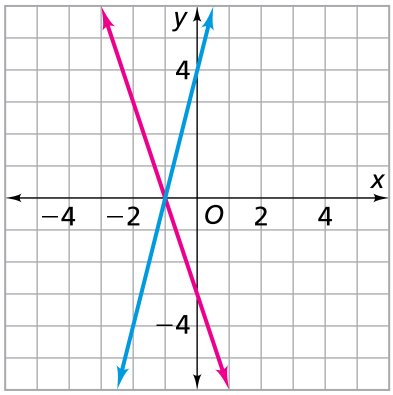

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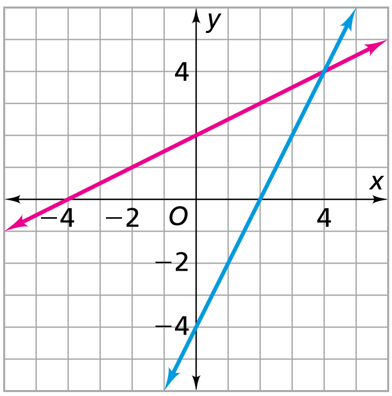

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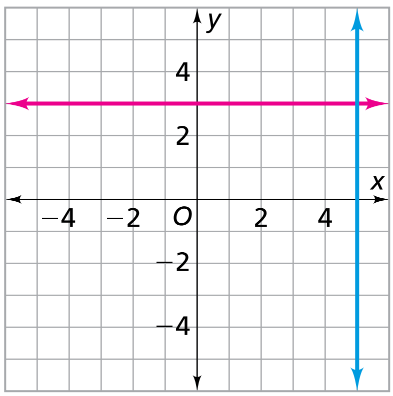


60. How does the solution of each system of equations in Exercises 56–59 relate to its the graph?

Write a system of equations that has each solution.

61. $x = 3,\ y = 2$ **62.** $x = 0,\ y = 0$

63. $x = -4,\ y = -2$ **64.** $x = \frac{1}{2},\ y = \frac{1}{4}$

For Exercises 65–68, tell whether the table represents a linear, quadratic, exponential, or inverse variation relationship. Write an equation for the relationship.

65.

x	0	1	2	3	4	5	6	7
y	0	−3	−4	−3	0	5	12	21

66.

x	−1	0	1	2	3	4	5	6
y	$\frac{1}{3}$	1	3	9	27	81	243	729

67.

x	1	3	4	6	9	10	12	18
y	2	8	11	17	26	29	35	53

68.

x	1	2	3	4	6	8	10	12
y	12	6	4	3	2	1.5	1.2	1

69. Use the tables of four linear equations.

Line *a*

x	−3	−2	−1	0
y	6	2	−2	−6

Line *b*

x	−3	−1	0	4
y	−6	−2	0	8

Line *c*

x	−3	1	0	3
y	6	2	3	0

Line *d*

x	4	6	8	10
y	5	6	7	8

What is the solution of the system of equations formed by

a. lines a and b?

b. lines a and c?

c. lines a and d?

d. lines b and c?

e. lines b and d?

f. lines c and d?

Solve each equation for x.

70. $5(x + 4) - 2x = 5 + 6x + 2x$

71. $2(x + 2) - 6x = 6x + 8 - 2x$

72. $x^2 - 7x + 12 = 0$

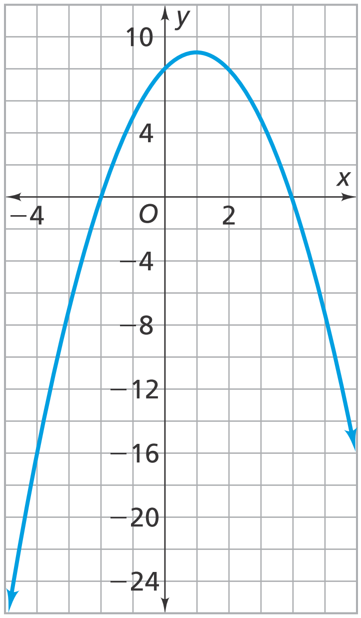
73. $x^2 + 5x - 6 = 0$

74. Match each equation with its corresponding graph below.

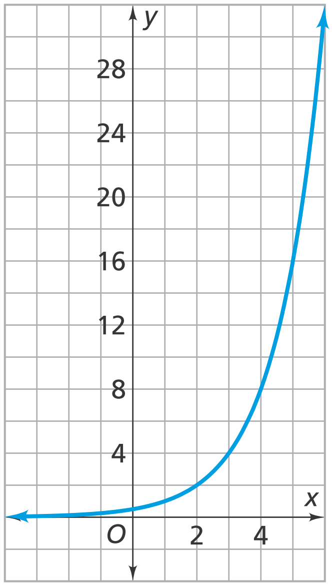
a. $y = 2^x - 1$ **b.** $y = -x^2 + 2x + 8$ **c.** $y = (x + 2)(x - 4)$

d. $y = 2^x$ **e.** $y = 2x^2$ **f.** $25 = x^2 + y^2$

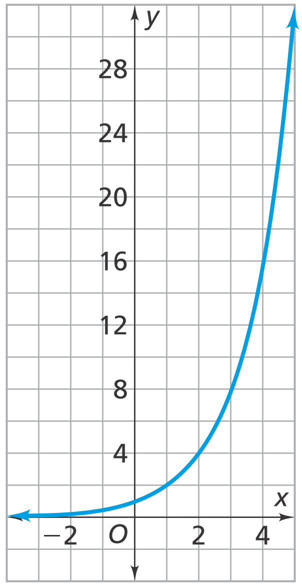
Graph 1



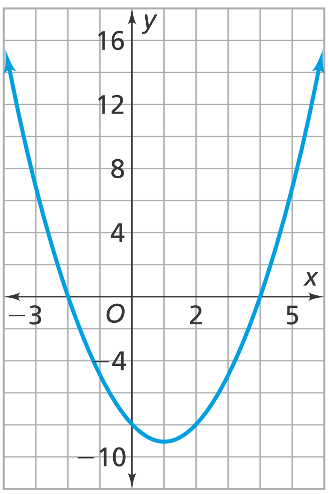
Graph 2



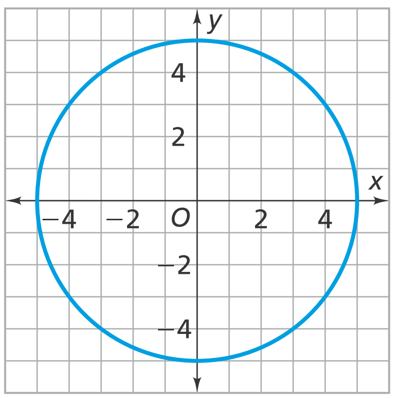
Graph 3



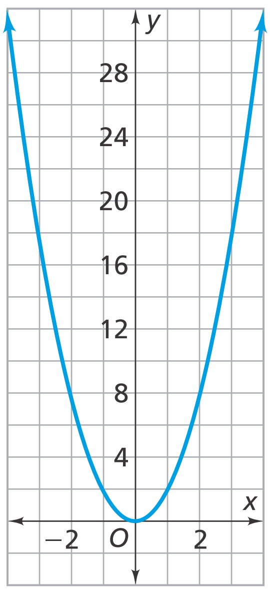
Graph 4



Graph 5



Graph 6



Extensions

75. Antonia and Marissa both babysit. Antonia charges \$5.50 an hour. Marissa charges a base rate of \$20.00, plus \$.50 an hour.

a. For each girl, write an equation showing how the charge depends on babysitting time.

b. For what times are Marissa's charges less than Antonia's?

c. Is there a time for which Antonia and Marissa charge the same amount?

76. Raj's age is 1 year less than twice Sarah's age. Toni's age is 2 years less than three times Sarah's age.

a. Suppose Sarah's age is s years. What is Raj's age in terms of s?

b. How old is Toni in terms of s?

c. How old are Raj, Sarah, and Toni if the sum of their ages is 21?

77. Melissa and Trevor sell candy bars to raise money for a class field trip. Trevor sells 1 more than five times as many candy bars as Melissa sells. Together they sell 49 candy bars.

a. Let m represent the number of candy bars Melissa sells. Let t represent the number of candy bars Trevor sells. Write a linear system to represent this situation.

b. Solve your system to find the number of candy bars each student sells.

78. Solve each system by writing each equation in the equivalent form $y = mx + b$ or by using the combination method. You may get some interesting results. In each case, graph the equations and explain what the results indicate about the solution.

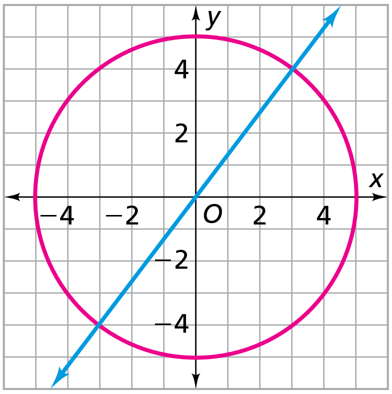
a. $\begin{cases} x - 2y = 3 \\ -3x + 6y = -6 \end{cases}$

b. $\begin{cases} x - y = 4 \\ -x + y = -4 \end{cases}$

c. $\begin{cases} 2x - 3y = 4 \\ 4x - 6y = 7 \end{cases}$

d. $\begin{cases} 4x - 6y = 4 \\ -6x + 9y = -6 \end{cases}$

79. The equation of the line is $y = \frac{4}{3}x$. The equation of the circle is $x^2 + y^2 = 25$.



You can find the intersection points by solving the system below. Modify the combination method to solve the system.

$$\begin{cases} y = \frac{4}{3}x \\ x^2 + y^2 = 25 \end{cases}$$

80. In Investigation 1, you learned that the solution of a system of linear equations is the intersection point of their graphs. Determine the maximum number of intersection points for the graphs of each type of function given.

a. a quadratic function and a linear function

b. a quadratic function and a different quadratic function

c. a cubic function and a linear function

d. an inverse variation and a linear function

e. Which pairs of functions in parts (a)–(d) might not have an intersection?

81. Write a system in the form $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$ that has the given solution.

a. (3, 7)

b. (−2, 3)

c. no solution

82. Consider these equivalent systems.

$$\begin{cases} 2y - 3x = 0 \\ y + x = 75 \end{cases} \quad \text{and} \quad \begin{cases} 2y - 3x = 0 \\ 3y + 3x = 225 \end{cases}$$

a. Do the four equations in these two systems represent four different lines? Explain.

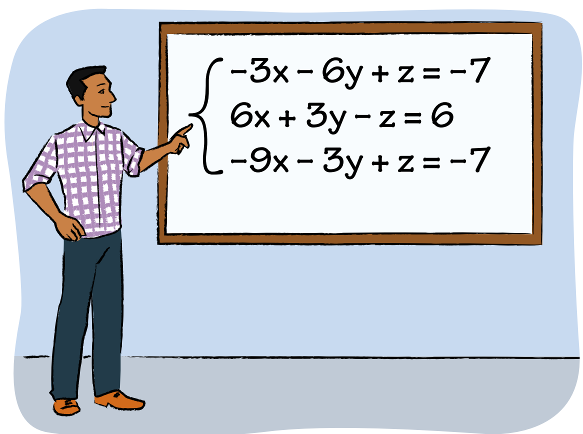
b. Adding the two equations in the second system gives $5y = 225$, or $y = 45$. Does $y = 45$ represent the same line as either equation in the system? Does its graph have anything in common with the lines in the system?

c. If you add the two equations in the first system, you get $3y - 2x = 75$. Does this equation represent the same line as either equation in the system? Does its graph have anything in common with the lines in the system?

d. What conjectures can you make about the results of adding any two linear equations? Consider the following questions:

- Will the result be a linear equation?
- Will the graph of the new equation have anything in common with the graphs of the original equations?

83. During math class, Mr. Krajewski gives Ben the following system.



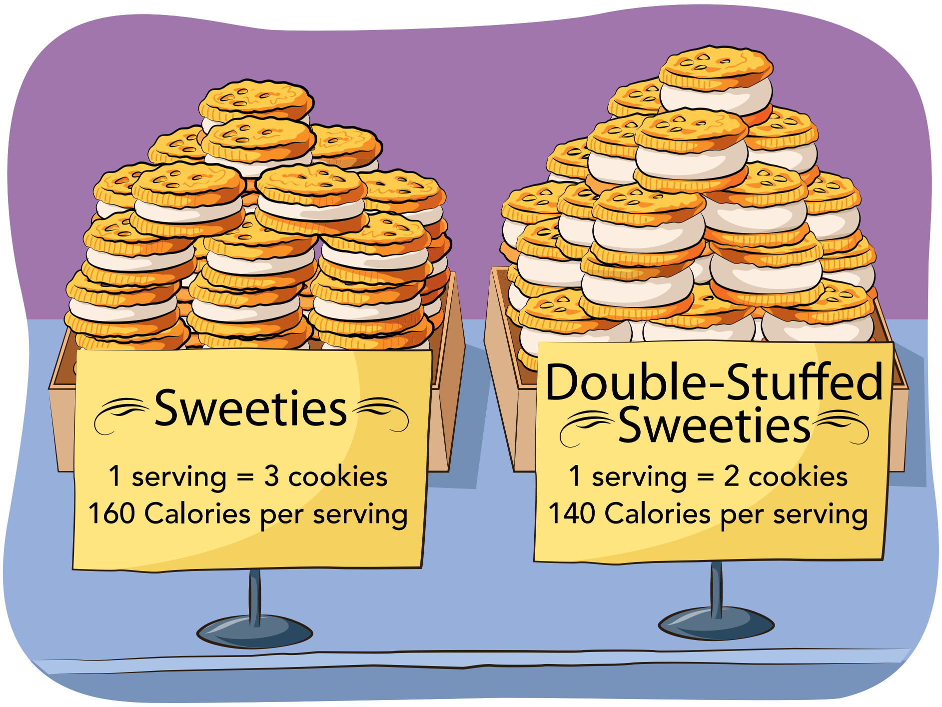
Ben thinks that he can solve the system if he adds the first equation to the second equation.

a. Why do you think Ben adds the two equations?

b. Is there another pair of equations Ben should add together?

c. Find the values of x, y, and z that satisfy the three equations.

84. A baking company makes two kinds of Sweeties, regular and double-stuffed.



a. How many wafers are in one serving of regular Sweeties? How many wafers are in one serving of double-stuffed Sweeties?

b. Let w represent the number of Calories in each wafer and f represent the number of Calories in each layer of filling.

i. What equation shows the relationship between w, f, and the number of Calories in one serving of regular Sweeties?

ii. What equation shows the relationship between w, f, and the number of Calories in one serving of double-stuffed Sweeties?

c. Solve the system of equations from part (b) to find the number of Calories in each Sweetie wafer and each layer of filling.