

**Applications**

1. Heidi's conjecture is correct; any value of  $1^x$  will always equal 1.

Evan's conjecture is correct; students might argue that it is the largest number in its row and column, so it will be the largest overall.

Jean's conjecture is incorrect, because  $2^3 = 8$ , however modifying the conjecture to have an odd numbered base would make it correct.

Chaska's conjecture is correct, because  $10^{n+1}$  is the same as  $10 \cdot 10^n$ .

Tim's conjecture is correct; any number raised to the second power is a square number.

Roger's conjecture is incorrect, because any number in this column has a 2 (actually many 2's) in its prime factorization, making it even.

2. B

3.  $40^6$

4.  $7^{15}$

5.  $8^5$

6. True; this is an example of  $a^m \times a^n = a^{m+n}$  and so  $6^3 \times 6^5 = 6^{3+5} = 6^8$ .

7. False;  $2^3 \times 3^2 = 8 \times 9 = 72$  and  $72 \neq 6^5$ .

8. True;  $3^8 = (3 \times 3)(3 \times 3)(3 \times 3)(3 \times 3) = (3^2)^4 = 9^4$ .

9. False;  $4^3 + 5^3 = 64 + 125 = 189$  and  $189 \neq 9^3$ .

10. True; by the Distributive Property,  $2^3(1 + 2^2) = (2^3 \times 1) + (2^3 \times 2^2) = 2^3 + 2^5$ . Or students may evaluate both sides and find that both sides are equal to 40.

11. False;  $\frac{5^{12}}{5^4} = 5^8 \neq 5^3$

12. H

13. 8; because  $4^{15} \times 3^{15} = (4 \times 3)^{15} = 12^{15}$ , the ones digit is the same as the units digit for  $2^{15} = 32,768$ .

14. 8; the ones digits for powers of 7 occur in cycles of 7, 9, 3, and 1. Because 15 divided by 4 leaves a remainder of 3, the ones digit of  $7^{15}$  is the third digit in the cycle which is 3. The ones digits for powers of 4 occur in cycles of 4 and 6. Because 20 is evenly divisible by 2, the ones digit of  $4^{20}$  is the second digit in the cycle, which is 6. So, the ones digit of  $7^{15} \times 4^{20}$  is the ones digit of  $3 \times 6 = 18$ .

15. a. Manuela is correct because  $2^{10} = 1,024$  and  $2^4 \times 2^6 = 16 \times 64 = 1,024$ .

- b. Possible answers:

$$2^2 \times 2^8 = 4 \times 256 = 1,024$$

$$2^3 \times 2^7 = 8 \times 128 = 1,024$$

$$2^2 \times 2^2 \times 2^6 = 4 \times 4 \times 64 = 1,024$$

- c. 4096; because  $2^7 = 128$  and  $2^5 = 32$ ,  $2^{12}$  would equal  $2^7 \times 2^5 = 128 \times 32 = 4,096$ .

- d. It works for other cases because you are just using the Associative Property of Multiplication. She is grouping strings of the same factor into two groups.

16. Yes; it has exactly 10 factors of 1.25.

17. Yes; it has exactly 10 factors of 1.25.

18. No;  $(1.25)^{10}$  is about 9.3 and  $(1.25) \times 10 = 12.5$ .

19. No;  $(1.25)^{10}$  is about 9.3 and  $(1.25) + 10 = 11.25$ .

20. Yes;  $(1.25^5)^2 = 1.25^5 \times 1.25^5$  which has exactly 10 factors of 1.25.

21. No;  $1.25^5 \times 1.25^5$  has exactly seven factors of 1.25, so it is equal to  $(1.25)^7$ , not  $(1.25)^{10}$ .

22. Yes; it has exactly 7 factors of 1.5

23. Yes; it has exactly 7 factors of 1.5.

24. No;  $(1.5)^7$  is about 17 and  $1.5 \times 7 = 10.5$ .

25. No;  $(1.5)^7$  is about 17 and  $1.5 + 7 = 8.5$ .

26. Both students are correct. For Stu, finding the fourth root might be easier first, because  $81^3$  is a somewhat large number. Carrie's solution is in many ways treating  $\frac{7}{3}$  as if it was a mixed number,  $2\frac{1}{3}$ .
27. 756; the expressions simplifies to  $756^1$ .
28. 342, because  $342^{\frac{5}{2}} \div 342^{\frac{3}{2}} = 342^{\frac{5-3}{2}} = 342^{\frac{2}{2}} = 342^1$ .
29. 1, because  $3^{35} \cdot 3^{-35} = 3^0 = 1$ .
30. 1, because  $(\frac{1}{2})^{40} = 2^{-40}$  so  $(\frac{1}{2})^{40} \cdot 2^{40} = 2^{-40} \cdot 2^{40} = 2^0 = 1$ .
31. Sometimes true, for example for  $n = 1$ , both sides equal 4. However, for  $n = 2$  the left side equals 16, and the right side equals 8.
32. Always true; any number times itself is the same as the square of the number.
33. Always false, because  $2^n = 2 \cdot 2^{n-1}$ .
34. Sometimes true. When  $b = 2$ , it is false as in Exercise 33. If  $b = \frac{1}{2}$ , then  $(\frac{1}{2})^n = \frac{1}{2}(\frac{1}{2})^{n-1}$ , so it is true for  $b = \frac{1}{2}$ .
35. Always true
36. Sometimes true; true when  $b > 1$  (as in part (e)), however, if  $0 < b < 1$ , then for a negative value of  $x$ ,  $b^x > 1$ .
- 37 a.  $p = 0.02(1.05)^n$   
 b. 1900:  $p = 0.02(1.05)^{33} = \$0.10$ ;  
 2000:  $p = 0.02(1.05)^{133} = \$13.16$   
 c. \$1: 1947; \$100: 2041  
 d. 1420 years later, or in the year 3287
38. a.  $n = \frac{1}{12}$ ,  $v = \$7,229,333.69$   
 b.  $n = \frac{1}{6}$ ,  $v = \$7,258,786.89$   
 c.  $n = \frac{1}{2}$ ,  $v = \$7,377,804.55$   
 d.  $n = \frac{5}{4}$ ,  $v = \$7,652,778.09$   
 e.  $n = \frac{5}{3}$ ,  $v = \$7,809,945.35$

Standard Form	Exponential Form
10,000	$10^4$
1,000	$10^3$
100	$10^2$
10	$10^1$
1	$10^0$
$\frac{1}{10} = 0.1$	$10^{-1}$
$\frac{1}{100} = 0.01$	$10^{-2}$
$\frac{1}{1,000} = 0.001$	$10^{-3}$
$\frac{1}{10,000} = 0.0001$	$10^{-4}$
$\frac{1}{100,000} = 0.00001$	$10^{-5}$
$\frac{1}{1,000,000} = 0.000001$	$10^{-6}$

- 39.
40. 0.3, 0.015, 0.0015
41. a. 0.12, 0.012, 0.0012, 0.000000012  
 b. Because  $1.2 \times 10^{-n} = 1.2 \times \frac{1}{10^n}$ , the standard form is 1.2 divided by the  $n$ th power of 10. When dividing by a power of 10, the decimal point in the number moves to the left. Because 1.2 is divided by  $n$ th power of 10, the decimal place is moved to the left  $n$  places; thus  
 $1.2 \times \frac{1}{10^n} = \underbrace{0.0000 \dots 0000012}_{n-1 \text{ zeros}}$
42. a.  $2,000,000 = 2 \times 10^6$   
 b.  $28,000,000 = 2.8 \times 10^7$   
 c.  $19,900,000,000 = 1.99 \times 10^{10}$   
 d.  $0.12489 = 1.2489 \times 10^{-1}$   
 e.  $0.0058421998 = 5.8421998 \times 10^{-3}$   
 f.  $0.0010201 = 1.0201 \times 10^{-3}$
43. a. Possible answer:  $\frac{1.5 \times 10^{-4}}{10^8}$   
 b. Possible answer:  $1.5 \times 10^{-4} \times 10^{-8}$

- 44. a.** 1,740,000 meters  
**b.**  $1.0795 \times 10^{-1} \text{ m} = 0.10795 \text{ m}$ .  
**c.** The scale that would make the image fit exactly is  $6.204 \times 10^{-8}$ . Any scale factor smaller than this will make the image small enough to fit.  
**d.** About 0.025 : 1
- 45.  $2^8 = 256$ , or  $2.56 \times 10^2$ .**  
**b.**  $\left(\frac{1}{2}\right)^8 = \frac{1}{256} = 2^{-8}$ ,  
 or  $0.00390625 = 3.90625 \times 10^{-3}$ .  
**c.**  $20^8 = 25,600,000,000$ , or  $2.56 \times 10^{10}$   
**d.**  $\left(\frac{1}{20}\right)^8 = \frac{1}{25,600,000,000} = 20^{-8}$   
 or  $0.000000000390625$ ,  
 or  $3.90625 \times 10^{-11}$
- 46. a.**  $3\frac{1}{3}, 1\frac{1}{9}, \frac{10}{27}, \frac{10}{81}, \frac{10}{243}, \frac{10}{729}, \frac{10}{2,187}$   
**b.** This means  $5.645029269 \times 10^{-5}$ .  
 In standard notation, it is 0.00005645029269.  
**c.**  $4.57 \times 10^{-3}, 1.52 \times 10^{-3}, 5.08 \times 10^{-4}, 1.69 \times 10^{-4}$
- 47.**  $\frac{1}{2}(2)^n = 2^{-1} \cdot 2^n = 2^{n-1}$
- 48.**  $4^{n-1} = 4^n \cdot 4^{-1} = 4^{-1} \cdot 4^n = \frac{1}{4}(4)^n$
- 49.**  $25(5^{n-2}) = 5^2 \cdot 5^{n-2} = 5^{n-2+2} = 5^n$
- 50. a.**  $4.10 \times 10^{11}$  gallons  $\times 365 \approx 1.496 \times 10^{14}$  gallons  
**b.**  $(1.28 \times 10^{11}) \div (2.14 \times 10^9) \approx 59.8$   
**c.**  $4.10 \times 10^{11} \times 0.80 \times 30 = 9.84 \times 10^{12}$
- 51.**  $1.722 \times 10^8$
- 52.**  $2.1 \times 10^{13}$
- 53.**  $2.8 \times 10^{-2}$
- 54.**  $3.5 \times 10^3$
- 55.**  $2.5 \times 10^{-2}$
- 56.**  $2 \times 10^{-8}$
- 57.**  $4 \times 10^4$
- 58.**  $g = 2.5, h = 9$
- 59.**  $j = 5, k = -1$
- 60.**  $m = 1.2, n = 2$
- 61.**  $p = 3, h = 8$
- 62.**  $r = 4, s = 2$
- 63.** The graphs of  $y = 4^x$  and  $y = 10(4^x)$  have the same growth factor of 4, so they are both exponential growth patterns. The graphs  $y = 0.25^x$  and  $y = 10(0.25^x)$  are exponential decay patterns and have the same decay factor of 0.25. The graphs of  $y = 4^x$  and  $y = 0.25^x$  have a y-intercept of (0,1). The graphs of  $y = 10(4^x)$  and  $y = 10(0.25^x)$  have y-intercepts (0,10).
- 64. a.** The graph is stretched vertically by a factor of 5. The y-values are all 5 times as far from zero (or 5 times greater than the original), though the growth factor is the same, 2.  
**b.** This is the same as  $10^x$ , so it should look like the graph is stretched vertically. The y-intercept remains the same, but the growth factor is now 5 times what it was.  
**c.** The  $\frac{1}{2}$  multiplies each y-value of the original graph, so it should reduce the values by a factor of  $\frac{1}{2}$ .  
**d.** This graph changes the sign of all the original y-values resulting in a flip over the x-axis.  
**e.** This graph flips the original graph over the y-axis.
- 65. a.** Graph B  
**b.** Graph C
- 66. a.** Grandville: 799; multiply 1000 by  $0.96^{5.5}$ .  
 Tintytown: 124; multiply 100 by  $1.04^{5.5}$ .  
**b.** Around 66 years;  $100 \cdot 1.04^{66} \approx 1331$ .  
**c.** Yes; the two towns will have the same populations if they continue to change at the same rates. Even though Grandville has a greater starting population, its population is decreasing, while Tintytown's population is increasing. So, eventually the graphs will cross. However, it will take about 28 years for this to happen.

**Connections**

- 67. 10 zeros
- 68. 50 zeros
- 69. 100 zeros
- 70. 6
- 71. 7

**Note:** Students may use their calculators for Exercises 72–74, but they should be able to use the rules of exponents and some estimation or mental arithmetic. The reasoning for  $6^9 > 9^6$ , for example, might look like this:

$$6^9 = (2 \times 3)^9 = 2^9 \times 3^9 = 2^9 \times 3^3 \times 3^6$$

and

$$9^6 = (3 \times 3)^6 = 3^6 \times 3^6 = 3^3 \times 3^3 \times 3^6$$

Comparing these comes down to comparing  $2^9$  and  $3^3$ . Because  $2^9 > 3^3$ ,  $6^9 > 9^6$ .

- 72.  $7^{10}$
- 73.  $8^{10}$
- 74.  $6^9$
- 75. False; since  $1.56892 \times 10^5 = 156,892$  is greater than  $2.3456 \times 10^4 = 23,456$ , the difference is greater than zero.
- 76. False; since  $3.96395 \times 10^5 = 396,395$  is less than  $2.888211 \times 10^7 = 28,882,110$ , the quotient is less than 1.
- 77. a. Volume:  $8 \text{ units}^3$ ; surface area:  $24 \text{ units}^2$ ; the side lengths increase to 2 units. The new volume is  $2^3 \text{ units}^3 = 2 \times 2 \times 2 \text{ units}^3 = 8 \text{ units}^3$ . Because there are six square faces, each with area  $2^2 \text{ units}^2 = 4 \text{ units}^2$ , the total surface area is  $6 \cdot 2^2 \text{ units}^3 = 24 \text{ units}^3$ .
- b. Volume:  $27 \text{ units}^3$ ; surface area:  $54 \text{ units}^2$ ; the side lengths increase to 3 units. The new volume is  $3^3 \text{ units}^3 = 3 \cdot 3 \cdot 3 \text{ units}^3 = 27 \text{ units}^3$ , and the new surface area is  $6 \cdot 3^2 \text{ units}^2 = 54 \text{ units}^2$ .

c. Volume:  $1,000,000 \text{ units}^3$ ; surface area:  $60,000 \text{ units}^2$ ; the side lengths increase to 100 units each. The new volume is  $100^3 \text{ units}^3 = 100 \cdot 100 \cdot 100 \text{ units}^3 = 1,000,000 \text{ units}^3$ , and the new surface area is  $6 \cdot 100^2 \text{ units}^2 = 60,000 \text{ units}^2$ .

- 78. a.  $8\pi \text{ units}^3$ ; the resulting cylinder has a radius of 2 units and a height of 2 units so the volume is  $\pi(2)^2 \times 2 \text{ units}^3 = 8\pi \text{ units}^3$ .
- b.  $27\pi \text{ units}^3$ ; the resulting cylinder has a radius of 3 units and a height of 3 units so the volume is  $\pi(3)^2 \times 3 \text{ units}^3 = 27\pi \text{ units}^3$ .
- c.  $1,000,000\pi \text{ units}^3$ ; the resulting cylinder will have a radius of 100 units and a height of 100 units, so  $V = \pi(100)^2 \times 100 \text{ units}^3 = 1,000,000\pi \text{ units}^3$ .
- 79. a. The following are prime:  $2^2 - 1 = 3$ ;  $2^3 - 1 = 7$ ;  $2^5 - 1 = 31$ .
- b. Other primes that fit this pattern include  $2^7 - 1 = 127$  and  $2^{13} - 1 = 8,191$ .
- 80. a. The sum of the proper factors of  $2^2$  is 3.
- b. The sum of the proper factors for  $2^3$ , or 8, is  $1 + 2 + 4 = 7$ .
- c. The sum of the proper factors for  $2^4$ , or 16, is  $1 + 2 + 4 + 8 = 15$ .
- d. The sum of the proper factors for  $2^5$ , or 32, is  $1 + 2 + 4 + 8 + 16 = 31$ .
- e. The sum of the proper factors of a power of 2 is always 1 less than the number.
- 81. a. Possible answer:  $\frac{3(10)^5}{10^7} = 3 \times 10^{-2} = 0.03 = \frac{3}{100}$ .
- b. Possible answer:  $\frac{5(10)^5}{25(10)^7} = 2 \times 10^{-3} = .002 = \frac{2}{1000} = \frac{1}{500}$ .

**Extensions**

- 82. 1; the ones digits for powers of 7 cycle through 7, 9, 3, and 1. Because the exponent, 100, is a multiple of 4, the ones digit will match the fourth number in the cycle, which is 1.
- 83. 6; the only possibility for the units digit for a power of 6 is 6.
- 84. 1; the ones digits for powers of 7 cycle through 7, 9, 3, and 1. The same is true for powers of 17. Because the exponent, 100, is a multiple of 4, the ones digit will match the fourth number in the cycle, which is 1.  $7^{100}$  and  $17^{100}$  have the same units digit.
- 85. 1; to get successive powers of 31, you repeatedly multiply by 31. The ones digit is always 1 times the previous ones digit. So the ones digit is always a power of 1, or 1.
- 86. 6; the possibilities for the ones digit when the base is 12 are the same as when the base is 2. So the ones digits cycle through 2, 4, 8 and 6. Because the exponent, 100, is a multiple of 4, the ones digit will be the fourth number in the cycle, which is 6.
- 87. 7; the possibilities for a include values with ones digits 3 or 7, because the ones digit in 823,543 is 3. Since 823,543 has 6 digits and the power is 7, 3 is too small, so a must equal 7. (17 or 27 or 37 etc. is too large.)
- 88. 11; a could be any number with a ones digit equal to 1, 3, 7 or 9. Since 1,771,561 has 7 digits and  $10^6 = 1,000,000$  has 7 digits, a must be greater than 10 but close to 10, so a is 11.
- 89. Possible answer: The ones digit is 9. The ones digits for the powers of 3 cycle through the pattern 3, 9, 7, 1, 3, 9, 7, 1 . . . So  $3^{28}$  will end in a 1,  $3^{29}$  will end in a 3, and  $3^{30}$  will end in 9.
- 90. C; square numbers have a ones digit of 1, 4, 9, 6, 5 or 0. So 1,392 is not a square number. However, 289 and 10,000 could be square numbers since they end in 0 and 9; in fact  $17^2 = 289$  and  $100^2 = 10,000$ .

- 91. a. Row 1:  $\frac{1}{2}$ , row 2:  $\frac{3}{4}$ , row 3:  $\frac{7}{8}$ , row 4:  $\frac{15}{16}$
- b.  $\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 = \frac{31}{32}$
- c.  $\frac{1,023}{1,024}, \frac{1,048,575}{1,048,576}$
- d. The sum of each row is a fraction with a denominator equal to 2 raised to the power of that row number, and a numerator that is 1 less than the denominator. In the  $n$ th row, the sum will be  $\frac{2^n - 1}{2^n}$ .
- e. Row 4
- f. 1
- g. The pattern is similar to adding the areas of one of the ballots produced by each cut. It may appear that this total area will eventually equal the area of the original sheet, but the pattern demonstrates that the total of the areas of the ballots will never actually equal the area of the whole piece.

92. a.

Number of Cuts	Area (ft <sup>2</sup> )
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$
5	$\frac{1}{32}$
6	$\frac{1}{64}$
7	$\frac{1}{128}$
8	$\frac{1}{256}$

- b.  $A = \left(\frac{1}{2}\right)^n$
- c. About  $9.54 \times 10^{-7}$  ft<sup>2</sup>. **Note:** this doesn't make sense, because a piece of paper could not be cut this small.

93. About \$71.42 per acre; the growth factor is 1.04. The cost has been increasing for 200 years ( $2003 - 1803 = 200$ ). Find the initial price per sq. mi:  
 $\$15,000,000 \div 828,000 \text{ sq. mi} \approx$   
 $\$18.13 \text{ per sq. mile.}$   
To get the initial price per acre, divide this value by 640:  $\$18.13 \text{ per sq. mile} \div$   
 $640 \text{ acres per sq. mile} \approx \$0.028 \text{ per acre.}$   
Thus, the value of 1 acre of land in 2006 is  
 $(\$0.028)(1.04)^{200} = (\$0.028)(2551) \approx \$71.42.$