Applications

1. Chen, from Problem 4.1, finds that his ballots are very small after only a few cuts. He decides to start with a larger sheet of paper. The new paper has an area of 324 in.^{2}. Copy and complete this table to show the area of each ballot after each of the first 10 cuts.

Areas of Ballots

Number of Cuts	Area (in.^{2})
0	324
1	162
2	81
3	■
4	■
5	■
6	■
7	■
8	■
9	■
10	■

a. Write an equation for the area A of a ballot after any cut n.

b. With the smaller sheet of paper, the area of a ballot is 1 in.^{2} after 6 cuts. Start with the larger sheet. How many cuts does it take to get ballots this small?

c. Chen wants to be able to make 12 cuts before getting ballots with an area of 1 in.^{2}. How large does his starting piece of paper need to be?

2. During the exploration of Problem 4.1, several groups of students in Mrs. Dole's class made a conjecture. They conjectured that the relationship between the number of cuts and the area of the ballot was an *inverse variation* relationship.

The class came up with two different arguments for why the relationship was not an inverse variation.

Argument 1

An inverse variation situation has a "factor-pair" relationship. Choose some constant number k. The two factors multiply to equal k, such as $yx = k$. For example, if the area of rectangle with length, l, and width, w, is 24,000 square feet, then $24{,}000 = lw$. This is an inverse variation.

In an exponential relationship, the values of the two variables x and y do not have this "factor-pair" relationship. For example, in Problem 4.1, the equation is $A = 64\left(\frac{1}{2}\right)^n$, but A and n do not multiply to get a constant number.

Argument 2

Any inverse variation will never have a y-intercept and this relationship does. Therefore, this relationship is not an inverse variation.

Which argument is correct? Explain why the students might have made this conjecture.

3. Latisha has a 24-inch string of licorice (LIK uh rish) to share with her friends. As each friend asks her for a piece, Latisha gives him or her half of what she has left. She doesn't eat any of the licorice herself.

a. Make a table showing the length of licorice Latisha has left each time she gives a piece away.

b. Make a graph of the data from part (a).

c. Suppose that, instead of half the licorice that is left each time, Latisha gives each friend 4 inches of licorice. Make a table and a graph for this situation.

d. Compare the tables and the graphs for the two situations. Explain the similarities and the differences.

4. Penicillin decays exponentially in the human body. Suppose you receive a 300-milligram dose of penicillin to combat strep throat. About 180 milligrams will remain active in your blood after 1 day.

a. Assume the amount of penicillin active in your blood decreases exponentially. Make a table showing the amount of active penicillin in your blood for 7 days after a 300-milligram dose.

b. Write an equation for the relationship between the number of days d since you took the penicillin and the amount of the medicine m remaining active in your blood.

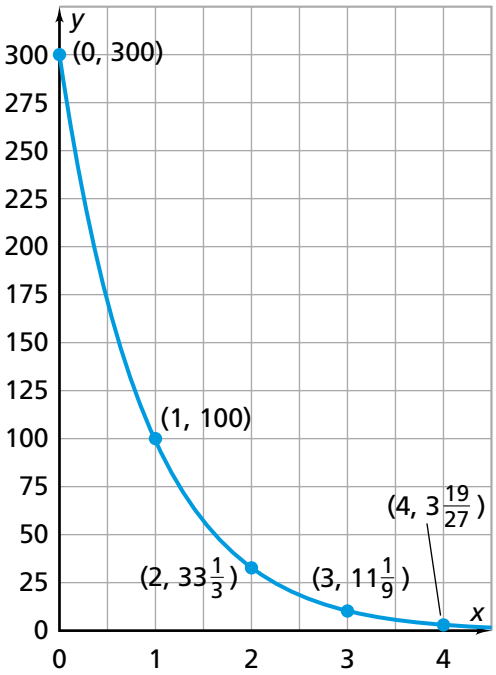
c. What is the equation for a 400-milligram dose?

For Exercises 5 and 6, tell whether the equation represents exponential decay or exponential growth. Explain your reasoning.

5. $y = 0.8(2.1)^x$

6. $y = 20(0.5)^x$

7. The graph below shows an exponential decay relationship.



a. Find the decay factor and the y-intercept.

b. What is the equation for the graph?

For Exercises 8 and 9, use the table of values to determine the exponential decay equation. Then, find the decay factor and the decay rate.

8.

x	*y*
0	24
1	6
2	1.5
3	0.375
4	0.09375

9.

x	*y*
0	128
1	96
2	72
3	54

For Exercises 10–13, use Lara's conjecture below. Explain how you found your answer.

> **Lara's Conjecture**
>
> If you know the y-intercept and another point on the graph of an exponential function, then you can find all the other points.

10. The exponential decay graph has y-intercept $= 90$, and it passes through $(2, 10)$. When $x = 1$, what is y?

11. The exponential decay graph has y-intercept $= 40$, and it passes through $(2, 10)$. When $x = 4$, what is y?

12. The exponential decay graph has y-intercept $= 75$, and it passes through $(2, 3)$. When $x = -2$, what is y?

13. The exponential decay graph has y-intercept $= 64$, and it passes through $(3, 0.064)$. When $x = 2$, what is y?

14. Karen shops at Aquino's Groceries. Her bill came to \$50 before tax. She used two of the coupons shown below.

Karen was expecting to save 10%, which is \$5. The cashier rang up the two coupons. Karen was surprised when the total price rang up as \$45.13 before tax. She was not sure why there was an extra \$0.13 charge.

a. What would explain why the coupons did not take off 10% the way Karen expected?

b. Write an equation to represent the total amount Karen would spend based on the number of coupons she would use.

c. Karen had originally thought that if she used 10 coupons on her next trip to Aquino's Groceries she would save 50%. Her bill is still \$50. How much would Karen actually spend?

d. How many coupons would you estimate it would take for Karen to get the \$50 of groceries for free?

15. Hot coffee is poured into a cup and allowed to cool. The difference between coffee temperature and room temperature is recorded every minute for 10 minutes.

Cooling Coffee

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Temperature Difference (°C)	80	72	65	58	52	47	43	38	34	31	28

a. Plot the data (*time, temperature difference*). Explain what the patterns in the table and the graph tell you about the rate at which the coffee cools.

b. Approximate the decay factor for this relationship.

c. Write an equation for the relationship between time and temperature difference.

d. About how long will it take the coffee to cool to room temperature? Explain.

16. The pizza in the ad for Mr. Costa's restaurant has a diameter of 5 inches.

a. What are the circumference and area of the pizza in the ad?

b. Mr. Costa reduces his ad to 90% of its original size. He then reduces the reduced ad to 90% of its size. He repeats this process five times. Extend and complete the table to show the diameter, circumference, and area of the pizza after each reduction.

Advertisement Pizza Sizes

Reduction Number	Diameter (in.)	Circumference (in.)	Area (in.^{2})
0	5	■	■
1	■	■	■

c. Write equations for the diameter, circumference, and area of the pizza after n reductions.

d. How would your equations change if Mr. Costa had used a reduction setting of 75%?

e. Express the decay factors from part (d) as fractions.

f. Mr. Costa claims that when he uses the 90% reduction setting on the copier, he is reducing the size of the drawing by 10%. Is Mr. Costa correct? Explain.

17. Answer parts (a) and (b) without using your calculator.

a. Which decay factor represents faster decay, 0.8 or 0.9?

b. Order the following from least to greatest:

0.9^4 0.9^2 90% $\frac{2}{10}$ $\frac{2}{9}$ 0.8^4 0.84

18. Natasha and Michaela are trying to find growth factors for exponential functions. They claim that if the independent variable is increasing by 1, then you divide the two corresponding y values to find the growth factor. For example, if (x_1, y_1) and (x_2, y_2) are two consecutive entries in the table, then the growth factor is $y_2 \div y_1$.

a. Is their reasoning correct? Explain.

b. Would this method work to find the growth pattern for a linear function? Explain.

Connections

For Exercises 19–22, write each number in scientific notation.

19. There are about 33,400,000,000,000,000,000,000 molecules in 1 gram of water.

20. There are about 25,000,000,000,000 red blood cells in the human body.

21. Earth is about 93,000,000 miles (150,000,000 km) from the sun.

22. The Milky Way galaxy is approximately 100,000 light years in diameter. It contains about 300,000,000,000 stars.

23. Consider these equations:

$$y = 0.75^x \qquad y = 0.25^x \qquad y = -0.5x + 1$$

a. Sketch graphs of all three equations on one set of coordinate axes.

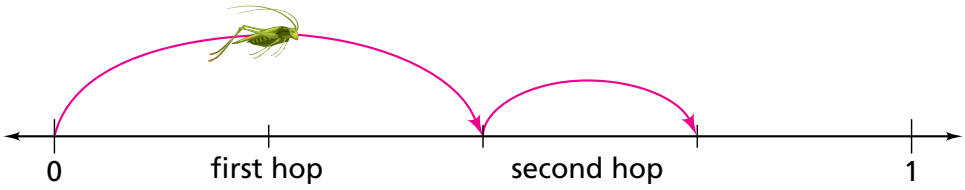
b. What points, if any, do the three graphs have in common?

c. In which graph does y decrease the fastest as x increases?

d. How can you use your graphs to figure out which of the equations is not an example of exponential decay?

e. How can you use the equations to figure out which is not an example of exponential decay?

24. A cricket is on the 0 point of a number line, hopping toward 1. She covers half the distance from her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.



a. Make a table showing the cricket's location for the first 10 hops.

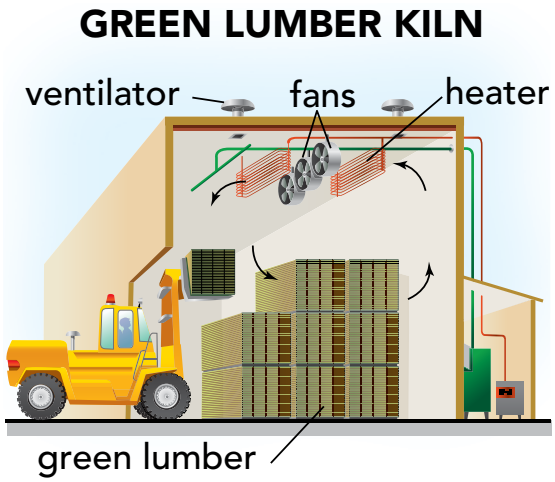
b. Where will the cricket be after n hops?

c. Will the cricket ever get to 1? Explain.

Extensions

25. Freshly cut lumber, known as *green lumber*, contains water. If green lumber is used to build a house, it may crack, shrink, and warp as it dries. To avoid these problems, lumber is dried in a kiln that circulates air to remove moisture from the wood.

Suppose that, in 1 week, a kiln removes $\frac{1}{3}$ of the moisture from a stack of lumber.



a. What fraction of the moisture remains in the lumber after 5 weeks in a kiln?

b. What fraction of the moisture has been removed from the lumber after 5 weeks?

c. Write an equation for the fraction of moisture m remaining in the lumber after w weeks.

d. Write an equation for the fraction of moisture m that has been removed from the lumber after w weeks.

e. Graph your equations from parts (c) and (d) on the same set of axes. Describe how the graphs are related.

f. A different kiln removes $\frac{1}{4}$ of the moisture from a stack of lumber each week. Write equations for the fraction of moisture remaining and the fraction of moisture removed after w weeks.

g. Graph your two equations from part (f) on the same set of axes. Describe how the graphs are related. How do they compare to the graphs from part (e)?

h. Green lumber is about 40% water by weight. The moisture content of lumber used to build houses is typically 10% or less. For each of the two kilns described above, how long should lumber be dried before it is used to build a house?