Applications

1. a.

Growth of Wolf Population

Year	Wolf Population
0	20
1	24
2	29
3	35
4	41
5	50
6	60

b. $p = 20(1.2)^t$, where p is the population and t is the number of years

c. about 9 years

2. a. The growth factor for the elk population is approximately 1.9. This is because
$$\frac{57}{30} \approx \frac{108}{57} \approx \frac{206}{108} \approx \frac{391}{206} \approx \frac{743}{391} \approx 1.9$$

b. After 10 years, there would be $30 \times (1.9)^{10} \approx 18,393$ elk. After 15 years, there would be $30 \times (1.9)^{15} \approx 455,434$ elk.

c. $p = 30 \times (1.9)^n$

d. After 16 years, the population is around 865,324 elk. After 17 years, the population is around 1,644,116 elk, so sometime between 16 and 17 years the population exceeds one million. Some industrious students might find by guess-and-check that the population exceeds one million after 16.225 years, or approximately 16 years and 3 months.

3. Between 1 and 2 years. $100(1.5) = 150$, and $100(1.5)^2 = 225$. Students may want to use a graph, a table, or guess-and-check to find a more precise answer: 1.71 years.

4. $p = 500,000 \times (1.6)^n$ (**Note:** This is not a good model as n gets large. In fact, in less than 500 years, it predicts there will be more squirrels than atoms in the universe.)

5. D

6. 1.3 years (**Note:** Students are likely to estimate the doubling time.)

7. 3.8 years (**Note:** Students are likely to estimate the doubling time.)

8. a.

$y = 50(2.2)^x$

x	0	1	2	3	4	5
y	50	110	242	532.4	1,171.3	2,576.8

$y = 350(1.7)^x$

x	0	1	2	3	4	5
y	350	595	1,011.5	1,719.6	2,923.2	4,969.5

b. In the first equation, the growth factor is 2.2. In the second, the growth factor is 1.7.

c. The graphs of these equations will cross because although the y-intercept of the first graph is lower, that graph is increasing at a faster rate.

d. The graphs will cross between $x = 7$ and $x = 8$. Some students might check carefully and find that the graphs cross at around $x = 7.547$.

9. a.

Maya's Savings Account

Age	Value
0	$100
1	$104
2	$108.16
3	$112.49
4	$116.99
5	$121.67
6	$126.53
7	$131.59
8	$136.86
9	$142.33
10	$148.02

b. 1.04

c. $a = 100(1.04)^n$, where a is the amount of money in the account and n is Maya's age.

10. 40%

11. 90%

12. 75%

13. 1.45

14. 1.9

15. 1.31

16. 1.25

17. a. 6 years; the projected population at that point is 1,340.

b. 6 years; the projected population at that time is 1,300. (**Note:** The linear equation $p = 1{,}000 + 50x$ models the problem, where p is the population in year x. Solving $1{,}300 = 1{,}000 + 50x$ shows that the population will outgrow the facilities in 6 years. You can also compare the two growth models by looking at tables for $y = 1{,}000(1.05)^x$ and $y = 1{,}000 + 50x$. This is particularly easy if you use a calculator to generate the tables. You might ask students to continue to scroll beyond the values for the first 6 years and see what they discover. Beyond that time, the exponential assumption will produce greater year-to-year growth.)

18. a.   **Radios Sold**

Year	Number of Radios
1	1,000,000
2	1,030,000
3	1,060,900
4	1,092,727
5	1,125,509
6	1,159,274
7	1,194,052

b.   **Radios Sold**

Year	Number of Radios
1	100,000
2	103,000
3	106,090
4	109,273
5	112,551
6	115,927
7	119,405

19. a. about \$8.72 (**Note:** The related equation is $p = 7(1.045)^t$, where p is the price of the ticket and t is the time in years; when $t = 5$, p is about \$8.72.)

b. about \$10.87 after 10 years; about \$26.22 after 30 years

c. 30 years

20. 100%

21. A. Choice A results in about 72% total growth over the 8 years. Choices B and C each give about the same total growth: 70%. Some students may find the amount at the end of each plan and compare the ending amounts.

22. a. initial value: \$130; growth rate: 7%, growth factor: 1.07; number of years: 5

b. \$223.36

23. Expressed as percents, the growth factors are Carlos: 114%; Mila: 125%; and Latanya: 300%.

a. Latanya's mice are reproducing most quickly.

b. Carlos's mice are reproducing most slowly.

Connections

24. $3,600

25. $300

26. $3,325

27. This pattern represents exponential growth because each value is the previous value times a growth factor of 1.1.

28. This pattern represents exponential growth because each value is the previous value times a growth factor of $\frac{5}{3}$.

29. This pattern does not represent exponential growth because there is no constant by which each value is multiplied to find the next value. The pattern is in fact linear, with an addition of $\frac{5}{3}$ for each term.

30. Answers may vary. A student could argue that the growth factor is approximately 3.2 and be correct. If this were "real world" data, most people (for most purposes) would consider this exponential growth. Another student might say that since there is variation in the growth factor between 3.18 and 3.22, this does not represent exponential growth.

31. **a.** 3% raise: $600; 4% raise: $800; 5% raise: $1,000

b. 3% raise: $20,600; 4% raise: $20,800; 5% raise: $21,000

c. Possible answer: Because 103% = 100% + 3%, 103% of $20,000 is the same as 100% of $20,000 plus 3% of $20,000. This means the same as $20,000 + (3% of $20,000). Or, since 103% = 1.03, you can reason as follows:

103% of $20,000 = 1.03($20,000)

= 1($20,000) + 0.03($20,000)

= $20,000 + 0.03($20,000)

= $20,000 + (3% of $20,000)

32. **a.** The bars represent the number of subscribers for each year.

b. The implied curve represents the pattern of growth in the total number of subscribers.

c. Answers will vary. At a glance, the pattern of change looks like exponential growth, but you cannot determine equal ratios from the graph. The 15-year growth is about 1300%, which gives an annual growth rate of about 19%. Note that the ratios of the numbers of subscribers in successive years appear to be approximately $\frac{30}{20}$, $\frac{40}{30}$, $\frac{49}{40}$, $\frac{60}{49}$, $\frac{75}{60}$, $\frac{95}{75}$, etc. These are not equal, so the pattern is not exactly an exponential function, but it can be approximated by an exponential function. It is not linear.

d. The growth between these two years is only about 7%, a growth factor of 1.07. This is significantly less than the growth factor in the preceding years.

e. One explanation is that many people are now using cell phones instead of landlines, and some people may have more than one subscription (for example, a business and a personal phone). Also, at some point, there may be a saturation point, when everyone has a cell phone.

33. a. The length of the diagonal is 5 cm, and the area of the shaded region is 6 cm^2.

b. For the enlargement, the length of the diagonal is 5.5 cm, and the area of the shaded region is 7.26 cm^2.

(**Note:** As each linear dimension increases by a factor of 1.1 (a 10% increase), the area increases by $(1.1)^2 = 1.21$.)

c. After five enlargements, the length of the diagonal is about 8.05 cm, and the area of the shaded region is about 15.56 cm^2.

d. Arturo is correct. For example, for the first few enlargements, the ratios are given here: $\frac{3}{4} = \frac{3.3}{4.4} = \frac{3.6}{4.84}$. The ratio is consistently $\frac{3}{4}$.

Esteban is incorrect because he is comparing a length measure to an area. Similarity requires a comparison of two length measures. If he had compared diagonal length to width, or diagonal length to length, Esteban would have been correct.

34. She is correct. If her salary the first summer is s, then under her plan, the second summer her salary will be $(1.04)s$ and $(1.03)(1.04)s$ the third summer, for a total of $s + (1.04)s + (1.03)(1.04)s$. Under the customer's plan, her total would be $s + (1.03)s + (1.04)(1.03)s$. Her earnings the third summer will be the same under both plans, but because she will make more money the second summer, her total earnings will be greater under her plan.

35. a. $\$9.00 \times 40 \times 52 = \$18,720$

b. $a = 360w$ (**Note:** Some students may include the paid vacation time and write the equation $a = 360w + 720$.)

c. She is trying to figure out how many weeks she needs to work in order to earn $9,000. The answer is 25 weeks.

d. **Kim's Salary**

Year	Annual Income
1	$18,720
2	$19,282
3	$19,860
4	$20,456
5	$21,070
6	$21,702
7	$22,353
8	$23,023
9	$23,714
10	$24,425

e. For the first 6 years, the $600-per-year raise plan is better. Under the $600-per-year plan, Kim would earn $21,720 in year 6 and $22,320 in year 7. In year 7, the salary for the 3% raise plan would be $22,353 and from then on would result in greater yearly salaries than the $600-per-year raise plan. The plan Kim chooses would depend on how many years she anticipates working for this company. (**Note:** Graphing the equations may not help students answer this question; for x-values from 0 to 10, both graphs look linear because the exponential growth is very slow for the first 10 years.)

36. 2.5 represents faster growth. 2.5 is greater than $25\% = \frac{1}{4}$. Note that the growth factor 2.5 represents the growth rate 1.5 or 150%. This is much faster than the growth rate 25%.

37. 130%, 1.475, $\frac{3}{2}$, 2

38. Answers may vary. Anything less than or equal to 88% (the scale factor that takes $8\frac{1}{2}$ to $7\frac{1}{2}$) will work.

39. a. Matches: 20% and 1.2; 120% and 2.2; 50% and 1.5; 400% and 5; 2% and 1.02. No match: 200%, 4, 2.

b. 2%, 20%, 50%, 120%, 200%, 400%

c. 1.02, 1.2, 1.5, 2, 2.2, 4, 5

Extensions

40. 2,500%. Because the growth factor is 26, the growth rate is $26 - 1$, or 25, expressed as a percent, which is 2,500%.

41. **a.** According to these assumptions, in 2020, the population would be about 344.09 million.

   b. about 70 years

   c. The actual growth rate for this time period was less than that predicted by the model in this exercise.

42. **a.** Averaging the ratios gives a growth factor of

$$\left(\frac{3.02}{2.76} + \frac{3.33}{3.02} + \frac{3.69}{3.33} + \frac{4.07}{3.69} + \frac{4.43}{4.07} + \frac{4.83}{4.43} + \frac{5.26}{4.83} + \frac{5.67}{5.26} + \frac{6.07}{5.67} + \frac{6.46}{6.07} + \frac{6.84}{6.46}\right) \div 11 \approx 1.1$$

   This is the growth factor for every 5 year increment, not for every 1 year.

   b. $p = 2.76(1.1)^x$, where x is the number of 5-year intervals.

   c. $p = 2.76(1.1)^8 \approx 5.92$ billion, so the population will double the 1955 population somewhere between 1990 and 1995 (the eighth 5-year period.)

   d. When $x = 17$, $p = 2.76(1.1)^{17} \approx 13.95$ billion, so the population will double the 2010 population somewhere between 2035 and 2040 (the seventeenth 5-year period). (**Note:** Doubling time is independent of the starting population.)

43. $p = 300(1.2)^t$, where p is the population and t is the year.

44. $p = 579(1.2)^t$, where p is the population and t is the year. Also, $p = 1,000(1.2)^{t-3}$ is acceptable.

45. $v = 2,413(1.03)^t$, where v is the value in the account and t is the year. Also, $v = 2,560(1.03)^{t-2}$ is acceptable.

46. **a.** Possible Answer: You could evaluate $(((1.5)^2)^2)^3$; in other words, multiply $1.5 \times 1.5 = 2.25$, then

   $2.25 \times 2.25 = 5.0625$, and then

   $5.0625 \times 5.0625 \times 5.0625 = 129.75$.

   b. You have to press the ☒ key 4 times to get the answer in the method outlined above.

47. **a.** $10,400

   b. $10,406.04

   c. $10,407.42. (**Note:** This is the exact answer using a growth factor of

   $1.00\overline{3} = \frac{1}{12}(0.04)$. However, students may round and use a growth factor of 1.003. This gives an answer of $10,366.00, which is significantly less. In compound growth situations, rounding leads to significantly different answers over time.)

   d. He will earn more if he chooses the account for which interest is compounded monthly. The more often the interest is compounded, the faster the investment grows.