Applications

1. If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose a single bacterium lands on your tooth and starts multiplying by a factor of 4 every hour.

a. Write an equation that describes the number of bacteria b in the new colony after n hours.

b. How many bacteria will be in the colony after 7 hours?

c. How many bacteria will be in the colony after 8 hours? Explain how you can find this answer by using the answer from part (b) instead of the equation.

d. After how many hours will there be at least 1,000,000 bacteria in the colony?

e. Suppose that, instead of 1 bacterium, 50 bacteria land in your mouth. Write an equation that describes the number of bacteria b in this colony after n hours.

f. Under the conditions of part (e), there will be 3,276,800 bacteria in this new colony after 8 hours. How many bacteria will there be after 9 hours and after 10 hours? Explain how you can find these answers without going back to the equation from part (e).

2. Loon Lake has a "killer plant" problem similar to Ghost Lake in Problem 2.1. Currently, 5,000 square feet of the lake is covered with the plant. The area covered is growing by a factor of 1.5 each year.

a. Copy and complete the table to show the area covered by the plant for the next 5 years.

b. The surface area of the lake is approximately 200,000 square feet. How long will it take before the lake is completely covered?

Growth of Loon Lake Plant

Year	Area Covered (sq. ft)
0	5,000
1	
2	
3	
4	
5	

3. Leaping Liang just signed a contract with a women's basketball team. The contract guarantees her \$20,000 the first year, \$40,000 the second year, \$80,000 the third year, \$160,000 the fourth year, and so on, for 10 years.

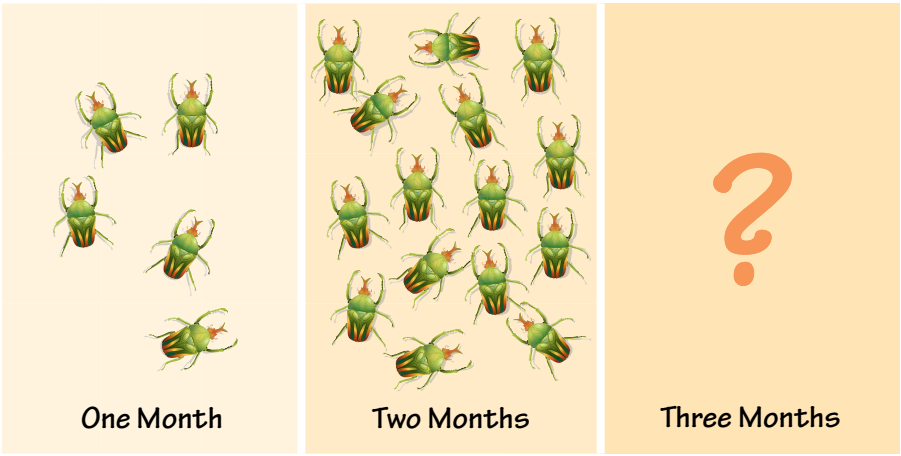
a. Make a table showing Liang's salary for each year of this contract.

b. What is the total amount Liang will earn over the 10 years?

c. Does the relationship between the number of years and salary represent an exponential function? Explain.

d. Write an equation for Liang's salary s for any year n of her contract.

4. As a biology project, Talisha is studying the growth of a beetle population. She starts her experiment with 5 beetles. The next month she counts 15 beetles.



a. Suppose the beetle population is growing linearly. How many beetles can Talisha expect to find after 2, 3, and 4 months?

b. Suppose the beetle population is growing exponentially. How many beetles can Talisha expect to find after 2, 3, and 4 months?

c. Write an equation for the number of beetles b after m months if the beetle population is growing linearly. Explain what information the variables and numbers represent.

d. Write an equation for the number of beetles b after m months if the beetle population is growing exponentially. Explain what information the variables and numbers represent.

e. How long will it take the beetle population to reach 200 if it is growing linearly?

f. How long will it take the beetle population to reach 200 if it is growing exponentially?

5. Fruit flies are often used in genetic experiments because they reproduce very quickly. In 12 days, a pair of fruit flies can mature and produce a new generation. The table below shows the number of fruit flies in three generations of a laboratory colony.

Growth of Fruit-Fly Population

Generations	0	1	2	3
Number of Fruit Flies	2	120	7,200	432,000

a. Does this data represent an exponential function? If so, what is the growth factor for this fruit-fly population? Explain how you found your answers.

b. Suppose this growth pattern continues. How many fruit flies will be in the fifth generation?

c. Write an equation for the population p of generation g.

d. After how many generations will the population exceed one million?

6. A population of mice has a growth factor of 3. After 1 month, there are 36 mice. After 2 months, there are 108 mice.

a. How many mice were in the population initially (at 0 months)?

b. Write an equation for the population after any number of months. Explain what the numbers and variables in your equation mean.

7. Fido did not have fleas when his owners took him to the kennel. The number of fleas on Fido after he returned from the kennel grew according to the equation $f = 8(3^n)$, where f is the number of fleas and n is the number of weeks since he returned from the kennel. (Fido left the kennel at week 0.)

a. How many fleas did Fido pick up at the kennel?

b. Is the relationship represented by the equation an exponential function? If so, what is the growth factor for the number of fleas?

c. How many fleas will Fido have after 10 weeks if they are untreated?

8. Consider the equation $y = 150(2^x)$.

a. Make a table of x and y-values for whole-number x-values from 0 to 5.

b. What do the numbers 150 and 2 in the equation tell you about the relationship between the variables x and y?

For Exercises 9–12, find the growth factor and the y-intercept of the equation's graph.

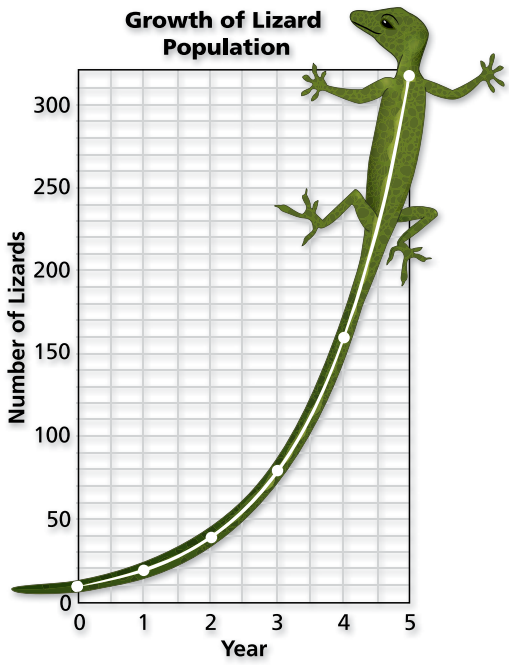
9. $y = 300(3^x)$

10. $y = 300(3)^x$

11. $y = 6{,}500(2)^x$

12. $y = 2(7)^x$

13. The following graph represents the population growth of a certain kind of lizard.



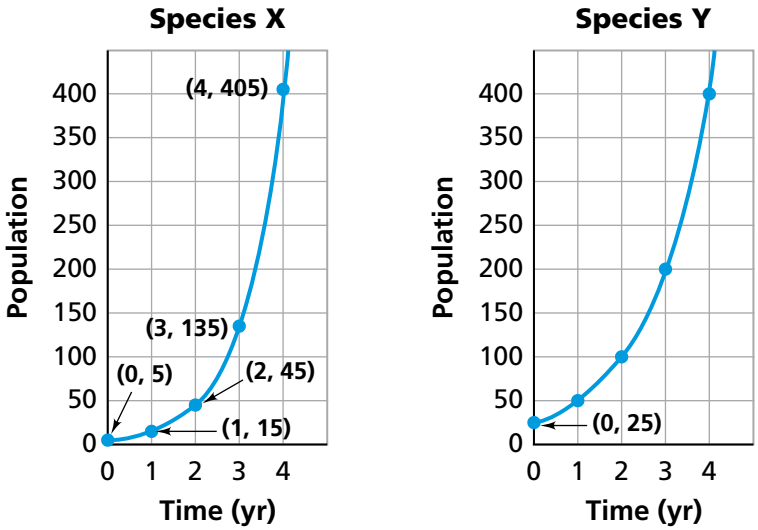
a. What information does the point (2, 40) on the graph tell you?

b. What information does the point (1, 20) on the graph tell you?

c. When will the population exceed 100 lizards?

d. Explain how you can use the graph to find the growth factor for the population.

14. The following graphs show the population growth for two species. Each graph represents an exponential function.



a. Find the growth factors for the two species.

b. What is the y-intercept for the graph of Species X? Explain what this y-intercept tells you about the population.

c. What is the y-intercept for the graph of Species Y? Explain what this y-intercept tells you about the population.

d. Write an equation that describes the growth of Species X.

e. Write an equation that describes the growth of Species Y.

f. For which equation is (5, 1215) a solution?

Connections

15. Multiple Choice Choose the answer that best approximates 3^{20} in scientific notation.

A. 3.5×10^{-9} **B.** 8×10^{3} **C.** 3×10^{9} **D.** 3.5×10^{9}

16. Multiple Choice Choose the answer that is closest to 2.575×10^{6}.

F. 21^{8} **G.** 12^{6} **H.** 6^{12} **J.** 11^{9}

17. Approximate 5^{11} in scientific notation.

For Exercises 18–20, decide whether each number is less than or greater than one million without using a calculator. Explain.

18. 3^6 **19.** 9^5 **20.** 12^6

For Exercises 21–23, write the prime factorization of each number using exponents. Recall the prime factorization of 54 is $3 \times 3 \times 3 \times 2$. This can be written using exponents as $3^3 \times 2$.

21. 45 **22.** 144 **23.** 2,024

24. Consider the two equations below.

Equation 1	Equation 2
$y = 10 - 5x$	$y = (10)5^x$

a. What is the y-intercept of each equation?

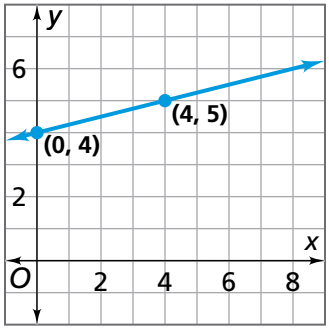
b. For each equation, explain how you could use a table to find how the y-values change as the x-values increase. Describe the change.

c. Explain how you could use the equations to find how the y-values change as the x-values increase.

d. For each equation, explain how you could use a graph to find how the y-values change as the x-values increase.

For Exercises 25–28, write an equation for each line. Identify the slope and y-intercept.

25.



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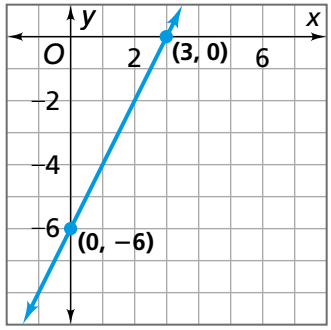

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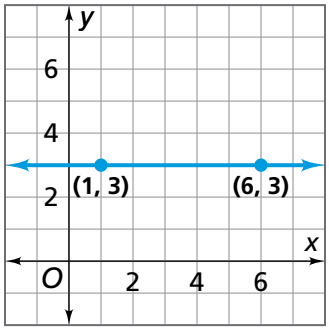

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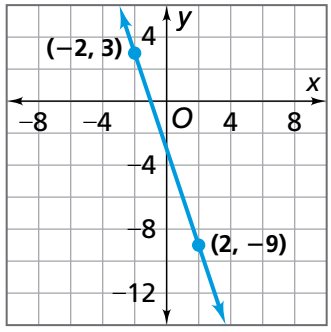
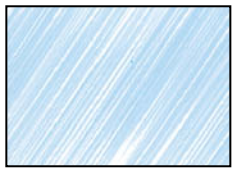


29. Maria enlarges a 2-cm-by-3-cm rectangle by a factor of 2 to get a 4-cm-by-6-cm rectangle. She then enlarges the 4-cm-by-6-cm rectangle by a factor of 2. She continues this process, enlarging each new rectangle by a factor of 2.



a. Copy and complete the table to show the dimensions, perimeter, and area of the rectangle after each enlargement.

Rectangle Changes

Enlargement	Dimensions (cm)	Perimeter (cm)	Area (cm^2)
0 (original)	2 by 3	■	■
1	4 by 6	■	■
2	■	■	■
3	■	■	■
4	■	■	■
5	■	■	■

b. Is the pattern of growth for the perimeter linear, exponential, or neither? Explain.

c. Does the pattern of growth for the area represent a linear function, exponential function, or neither? Explain.

d. Write an equation for the perimeter P after n enlargements.

e. Write an equation for the area A after n enlargements.

f. How would your answers to parts (a)–(e) change if the copier were set to enlarge by a factor of 3?

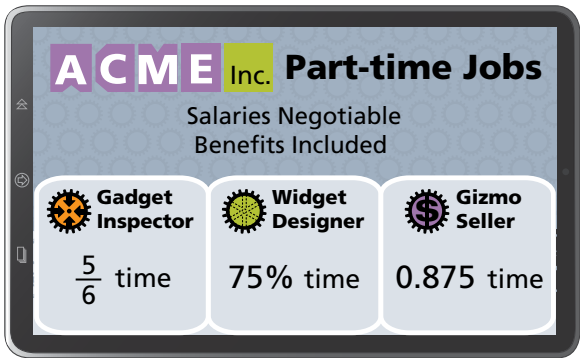
For Exercises 30 and 31, Kele enlarged the figure below by a scale factor of 2. Ahmad enlarged the figure 250%.

30. Who made the larger image?

31. Multiple Choice Which factor would give an image between Ahmad's image and Kele's image in size?

A. $\frac{2}{5}$ **B.** $\frac{3}{5}$ **C.** $\frac{9}{4}$ **D.** $\frac{10}{4}$

32. Companies sometimes describe part-time jobs by comparing them to full-time jobs. For example, a job that requires working half the number of hours of a full-time job is described as a $\frac{1}{2}$-time job or a 50%-time job. ACME, Inc. has three part-time job openings.



Order these jobs from the most time to the least time.

Extensions

33. **a.** Make a table and a graph for the equation $y = 1^x$.

b. How are the patterns in the table and the graph of $y = 1^x$ similar to patterns you have observed for other exponential and linear functions? How are they different?

34. If you know that a graph represents an exponential function, you can find the equation for the function from two points on its graph. Find the equation of the exponential function whose graph passes through each pair of points. Explain.

a. (1, 6) and (2, 12) **b.** (2, 90) and (3, 270)

35. Leaping Liang plays basketball. A team promised her \$1 million a year for the next 25 years. The same team offered Dribbling Dinara \$1 the first year, \$2 the second year, \$4 the third year, \$8 the fourth year, and so on, for 25 years.

a. Suppose Liang and Dinara each accept the offers and play for 20 years. At the end of 20 years, who receives more money?

b. Tell which player will receive more after 21 years, 22 years, 23 years, and 25 years.

c. Do either of the two plans represent an exponential function? Explain.