Applications

1. a. $b = 4^n$
   b. $4^7 = 16{,}384$ bacteria
   c. 65,536; this can be found by computing $16{,}384 \cdot 4$ because $4^8 = 4^7 \times 4$.
   d. 10 hours. There will be at least 1 million bacteria in the colony after 9 hr and before 10 hr, as shown by $4^9 = 262{,}144$ and $4^{10} = 1{,}048{,}576$. (**Note:** This is essentially solving the equation $1{,}000{,}000 = 4^n$. Students can solve this problem in a variety of ways. They might guess and check values of n in 4^n.They might make a chart. They might enter the equation into a calculator and look at the table. They might trace a calculator graph, although setting an appropriate graphing window for exponential equations can be challenging.)
   e. $b = 50(4^n)$
   f. There will be 13,107,200 bacteria after 9 hr and 52,428,800 after 10 hr. We can find these by multiplying the number of bacteria at 8 hours by 4, and then multiplying that number by 4.

2. a. **Loon Lake Plant Growth**

Year	Area Covered (ft^2)
0	5,000
1	7,500
2	11,250
3	16,875
4	25,312.5
5	37,968.75

   b. 10 yr, actually slightly more than 9 years

3. a. **Leaping Lenora's Salary**

Year	Salary
1	$20,000
2	$40,000
3	$80,000
4	$160,000
5	$320,000
6	$640,000
7	$1,280,000
8	$2,560,000
9	$5,120,000
10	$10,240,000

   b. $20,460,000; **Note:** Students can find this by adding the amounts in the table or by using their calculators to find the sum of the sequence of $S = 10{,}000(2^n)$ from $n = 1$ to 10.
   c. Yes, the relationship is an exponential function because the growth pattern is doubling from year to year.
   d. $s = 20{,}000(2^{n-1})$ or $s = 10{,}000(2^n)$

4 a. 25 beetles; 35 beetles; 45 beetles
   b. 45 beetles; 135 beetles; 405 beetles
   c. $b = 5 + 10m$, where b is the number of beetles and m is the number of months
   d. $b = 5(3^m)$ or $b = 15(3^{m-1})$, where b is the number of beetles and m is the number of months.
   e. 19.5 months; solve $200 = 5 + 10m$.
   f. Between 3 and 4 months; there are 135 beetles after 3 months and 405 beetles after 4 months. (**Note:** Students won't be able to solve the exponential equation algebraically. They can find an approximate solution by scrolling through a calculator table for the equation, using appropriate increments. Or, students might graph the equation and trace its graph.)

5. **a.** Yes; 60; the number of fruit flies in any generation divided by the number in the previous generation is 60.

   b. 1,555,200,000;

   $432{,}000 \times 60 \times 60 = 1.5552 \times 10^9$

   c. $p = 2(60^g)$

   d. 4

6. **a.** 12 mice; There were 36 mice after 1 month and the growth factor is 3. So at 0 months, there were $36 \div 3 = 12$ mice.

   b. $p = 12(3^n)$. 12 is the original population, 3 is the growth factor, p is the population, n is the number of months. (Or, $p = 36(3^{n-1})$, where 36 is the population after 1 month.)

7. **a.** 8 fleas

   b. Yes, the relationship is exponential. The growth factor is 3.

   c. $8(3^{10}) = 472,392$ fleas

   Note: Point out to students that this answer demonstrates that exponential growth equations have limits as models of real-life phenomena. Although something might start out increasing in a nearly exponential way, the predictive validity of the model will eventually break down.

8. **a.**

x	y
0	150
1	300
2	600
3	1,200
4	2,400
5	4,800

   b. The starting population, or initial value, is 150, and the growth factor is 2.

9. growth factor: 3; y-intercept: 300

10. growth factor: 3; y-intercept: 300

11. growth factor: 2; y-intercept: 6,500

12. growth factor: 7; y-intercept: 2

13. **a.** After 2 yr, the lizard population was 40.

    b. After 1 yr, the lizard population was 20.

    c. between years 3 and 4

    d. Divide the population for one year by the population for the previous year. For example, divide the population for year 3, which is 80, by the population for year 2, which is 40: $80 \div 40 = 2$.

14. **a.** The growth factor for Species X is 3 because the y-value for each point is 3 times the previous y-value.
    The growth factor for Species Y is 2 because the y-value for each point is 2 times the previous y-value.

    b. The y-intercept is (0, 5), so the starting population for Species X is 5.

    c. The y-intercept is (0, 25), so the starting population for Species Y is 25.

    d. $y = 5(3^x)$

    e. $y = 25(2^x)$

    f. (5, 1,215) is a solution for Species X. If we substitute 5 for x into each equation, we get 1,215 for Species X.

Connections

15. D

16. G

17. 4.88×10^7

18. Less than; 1 million is 10^6 and $3 < 10$. Therefore, $3^6 < 10^6$.

19. Less than; 1 million is 10^6 and $9 < 10$. Therefore, $9^5 < 9^6 < 10^6$.

20. Greater than; 1 million is 10^6 and $10 < 12$. Therefore, $10^6 < 12^6$.

21. $3^2 \times 5$

22. $2^4 \times 3^2$

23. $2^3 \times 11 \times 23$

24. a. The *y*-intercept is (0,10) for each equation.

b. If you make a table of (*x*, *y*) values for Equation 1 for consecutive *x*-values, you will see that the *y*-values decrease by 5, so the rate of change is −5. In the table for Equation 2, the values increase. If you subtract successive *y*-values, you get differences of 40, 200, 1,000, and so on. So the rate of change is increasing. (Students will learn in Investigation 3 that the growth rate is 400%.) (**Note:** Students may describe the pattern of change for Equation 2 multiplicatively, saying that each *y*-value is 5 times the previous *y*-value. You could ask these students to describe the change additively, which will get at the increasing rate of change described above.)

c. In Equation 1, the rate of change (the slope) is the −5 in front of the *x*. In the second equation the rate of change increases. Y will be growing 5 times as fast between $x = 2$ and $x = 3$ as it grew between $x = 1$ and $x = 2$. It is easier to see in a table. However, the growth factor of 5 can be seen in the equation as the number raised to the exponent. (**Note:** Students will be introduced to rate of change of exponential equations in Investigation 3, so this problem is just to get them to think about patterns of change for linear and exponential functions.)

d. Look at the vertical distance between points for each horizontal change of 1 unit. In the graph of Equation 1, the vertical distance between any two points is 5. In the graph of Equation 2, the vertical distance increases, indicating that the *y*-values are increasing at a faster and faster rate. Students may also describe this multiplicatively at this time. That is, each *y*-value is increasing 5 times the previous *y*-value.

25. $y = \frac{1}{4} + 4$; slope is $\frac{1}{4}$, *y*-intercept is (0, 4)

26. $y = 2x - 6$; slope is 2, *y*-intercept is (0, −6)

27. $y = 3$; slope is 0, *y*-intercept is (0, 3)

28. $y = -3x - 3$; slope is −3, *y*-intercept is (0, −3)

29. a.

Enlargement	Dimensions (cm)	Perimeter (cm)	Area (cm^2)
0 (original)	2 by 3	10	6
1	4 by 6	20	24
2	8 by 12	40	96
3	16 by 24	80	384
4	32 by 48	160	1,536
5	64 by 96	320	6,144

b. Exponential; each perimeter is multiplied by 2 to obtain the next perimeter.

c. Exponential; each area is multiplied by 4 to obtain the next area. (**Note:** Because both width and length increase by a factor of 2, area increases by a factor of 4.)

d. $P = 10(2^n)$

e. $A = 6(4^n)$ or $A = 3(2^n) \times 2(2^n)$

f. Perimeter and area would still increase exponentially, but the related equations would be $P = 10(3^n)$ and $A = 6(9^n)$.

30. Ahmad; expressed as a percent, Kele's scale factor is 200%, which is less than 250%.

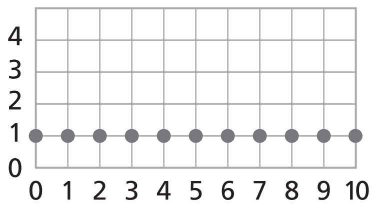
31. C

32. gizmo seller, gadget inspector, widget designer

Extensions

33. a.

x	y
0	1
1	1
2	1
3	1
4	1



b. The equation $y = 1^x$ looks like other exponential equations, but the pattern in the table—in which every value of 1^x is 1—and in the straight-line graph looks like a linear relationship.

34. a. $y = 3(2)^x$; the growth factor can be found by dividing the y-values: $12 \div 6 = 2$. The y-intercept can be found by dividing the y-value for $x = 1$, which is 6, by the growth factor of 2. So the y-intercept is (0, 3).

b. $y = 10(3)^x$; the growth factor can be found by dividing the y-values: $270 \div 90 = 3$. The y-intercept can be found by dividing the y-value for $x = 1$, which is 30, by the growth factor of 3. So the y-intercept is (0, 10).

35. a. Liang; at the end of 20 years, Liang would have $\$1{,}000{,}000(20) = \$20{,}000{,}000$, and Dinara would have $2^{20} - 1 = \$1{,}048{,}575$. Students will probably sum up the values for each year to find Dinara's total: $\$1 + \$2 + \ldots + \$2^{19} = \$1{,}048{,}575$. **Note:** Dinara receives dollars in salary, where n is the year number, and her total for the n years is $2^n - 1$.

b. Liang will continue to have a greater salary through year 25, when Dinara will overtake her with $33,554,431 to Liang's $25,000,000. **Note:** You may want to discuss with students a realistic time span for players in professional basketball. It is unusual for players to remain in high demand for 20 years or more. Salaries may even decrease after time.

c. Yes, Dinara's plan is exponential because the growth factor is 2. Leaping Liang's is not exponential because the growth rate is 1 (she gets the same amount of money each year).