GGG Inv 1-2 Topics

Review all Problems in Inv 1 and 2 including ACE

Know how to:

- Create a table and a graph of an exponential relationship given a description or equation
- Write an exponential equation given a graph, table, or two points
- Write expressions in exponential, expanded, and standard form
- Write numbers in scientific notation and standard form
- Write an exponential equation with a y-intercept other than 1
- Identify whether a table is linear, exponential, or neither based on a table, graph or equation
- Write the equation of a linear or exponential relationship given a table, graph
- Estimate when an exponential relationship will reach a certain number

1. Bamboo can grow rapidly. When it is very young, it can triple in size in a week. You start tracking how quickly a piece of bamboo grows when it is already 5 millimeters tall.

   Write an equation showing how the height of the bamboo changes as the number of weeks increases. **Explain what the variables and numbers in your equation mean in the context of this situation.** There should be 5 parts to your answer: equation, what your two variables mean, and what your two numbers mean.

   $y = 5(3)^x$

   - y: The height of the bamboo (mm) on Day x of growth
   - x: # of weeks the bamboo has been growing.
   - 5: The bamboo was 5mm tall on day zero of measurement.
   - 3: The height of the bamboo triples each week.

   How tall would the bamboo be after twelve weeks? Is this a reasonable estimate? Why or why not?

   $y = 5(3)^{12} = 2,657,205$ mm

   This is not reasonable. At some point the growth of a plant slows down.

2. Your friend also has some bamboo and has been tracking the growth over the last few weeks. Their table is below.

Weeks	3	4	5	6	7
Height (mm)	32	128	512	2048	8192

   (Weeks: +1 each step; Height: ×4 each step)

   Write an equation that models the growth of your friend's bamboo. Whose bamboo is growing at a faster rate?

Weeks	0	1	2	3	4	5	6	7
Height (mm)	0.5	2	8	32	128	512	2048	8192

   (Working backwards: −1 weeks, ÷4 height)

   $y = 0.5(4)^x$

   Using Algebra

   $y = a(4)^x$

   $\frac{32}{4^3} = \frac{a(4)^3}{4^3}$

   $0.5 = a$

1. A bathtub is being filled at a rate of 2.5 gallons per minute. The bathtub will hold 20 gallons of water.
   a. How long will it take to fill the bathtub?

$y = 2.5x$

$\frac{20}{2.5} = \frac{2.5x}{2.5}$

$8 = x$

It will take 8 minutes to fill the bathtub

   b. Is the relationship described linear, inverse, exponential, or neither? Write an equation relating the variables.

Linear $y = 2.5x$

x = # of minutes
y = # of gallons in the tub

2. Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 4 every hour.
   a. After how many hours will there be at least 1,000,000 bacteria in the new colony?

$y = 1(4)^x$

$4^{10} = 1,048,576$

It will take 10 hours.

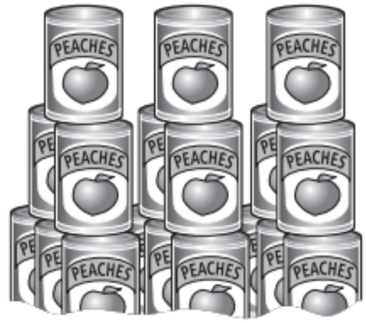
   b. Is the relationship described linear, inverse, exponential, or neither? Write an equation relating the variables.

$y = 4^x$

y = # of bacteria
x = # of hours growing

This relationship is EXPONENTIAL because as the value of x increases by 1, the value of y is multiplied by a constant factor of 4.

3. Two students who work in a grocery store are making a display of canned goods. They build a tower of cans 12 layers deep. The first layer, at the top, contains three cans in a row. The second layer contains six cans, in two rows of three that support the first layer. The third layer has nine cans, in three rows of three that support the second layer.



layer 1

layer 2

Layer	# of cans
1	3
2	6
3	9

(+1, +1 ; +3, +3)

a. How many cans are in layer 12, the bottom layer?

36 cans

b. Is the relationship described linear, inverse, exponential, or neither? Write an equation relating the variables.

$y = 3x$

4. An experimental plant has an unusual growth pattern. On each day, the plant doubles its height of the previous day. On the first day of the experiment, the plant grows to twice, or 2 times, its original height. On the second day, the plant grows to 4 times its original height. On the third day, the plant grows to 8 times its original height.

a. How many times its original height does the plant reach on the sixth day? On the *n*th day?

$2^6 = 64$ times as high

Height $= 2^n$

b. If the plant is 128 centimeters tall on the ninth day, how tall was it just before the experiment began?

$\frac{128}{2^9} = \frac{a(2^9)}{2^9}$

$0.25 = a$

The plant was 0.25" tall on day 0.

c. Is the relationship described linear, inverse, exponential, or neither? Write an equation relating the variables.

$y = 0.25(2)^n$

Decide whether the relationship is linear, exponential, or neither. If it is linear or exponential, write an equation for the relationship. If it is neither, explain why and be specific.

7.)

x	2	3	4	5
y	179	223	267	311

+1 +1 +1

+44 +44 +44

$\frac{\Delta y}{\Delta x} = \frac{44}{1}$

$y = 44x + b$

$179 = 44(2) + b$

$179 = 88 + b$

$-88 \quad -88$

$91 = b$

$y = 44x + 91$

8.)

x	1	2	3	4
y	16	32	64	256

+1 +1 +1

×2 ×2 ×4

Neither

This cannot be exponential because as x increases by 1, the y-value is not being multiplied by a constant amount.

9.)

x	3	4	5	6
y	24	48	96	192

+1 +1 +1

×2 ×2 ×2

$y = a(2)^x$

$\frac{24}{2^3} = \frac{a(2^3)}{2^3}$

$3 = a$

$y = 3(2)^x$

10.)

x	0	1	2	3	4
y	4.5	13.5	40.5	121.5	364.5

+1 +1 +1

×3 ×3 ×3

$y = a(3)^x$

$\frac{13.5}{3} = \frac{a(3^1)}{3}$

$4.5 = a$

$y = 4.5(3)^x$

Write an equation that best models the situation.

37.) A population of bugs has a growth factor of 4. After year 2, there are 480 bugs. After year 3, there are 1,920 bugs.

$y = a(4)^x$

$\frac{480}{4^2} = \frac{a(4^2)}{4^2}$

$30 = a$

0	1	2	3
30	120	480	1920

$\frac{1920}{480} = 4$

$y = 30(4)^x$

38.) A plant is growing exponentially on a lake. At present, it covers 300 square feet. After 1 month, it covers 450 square feet.

$\frac{450}{300} = 1.5$

$y = 300(1.5)^x$

39.) Mold is growing exponentially on an old piece of bread. After day 2, 4.5 square centimeters are covered. After day 3, 13.5 square centimeters are covered.

$\frac{13.5}{4.5} = 3$

$y = a(3)^x$

$\frac{4.5}{3^2} = \frac{a(3)^2}{3^2}$

$0.5 = a$

$y = 0.5(3)^x$

For the problems below, first assume a linear relationship and write the equation of the line passing between the points. Then, assume an exponential relationship and write the equation of the exponential curve passing between the points.

11.) (3, 12) and (4, 24)

Linear: $\frac{\Delta y}{\Delta x} = \frac{12}{1}$

$y = 12x + b$

$12 = 12(3) + b$

$12 = 36 + b$

$-36 \quad -36$

$-24 = b$

$y = 12x - 24$

Exponential:

$y = a(2)^x$

$\frac{12}{2^3} = \frac{a(2)^3}{2^3}$

$1.5 = a$

$y = 1.5(2)^x$

12.) (1, 8) and (2, 24)

Linear: $\frac{\Delta y}{\Delta x} = \frac{16}{1}$

$y = 16x + b$

$8 = 16(1) + b$

$-16 \quad -16$

$-8 = b$

$y = 16x - 8$

Exponential: $\frac{24}{8} = 3$

$y = a(3)^x$

$\frac{8}{3^1} = \frac{a(3^1)}{3^1}$

$\frac{8}{3} = a$

$y = \frac{8}{3}(3)^x$

13.) (3, 20) and (4, 80)

Linear: $\frac{\Delta y}{\Delta x} = 60$

$y = 60x + b$
$20 = 60(3) + b$
$20 = 180 + b$
$-180 \quad -180$
$-160 = b$

$y = 60x - 160$

Exponential:

$y = a(4)^x$
$\frac{20}{4^3} = \frac{a(4)^3}{4^3}$
$0.3125 = a$

$y = 0.3125(4)^x$

14.) (1, 18) and (2, 27)

Linear: $\frac{\Delta y}{\Delta x} = 9$

$y = 9x + b$
$18 = 9(1) + b$
$18 = 9 + b$
$-9 \quad -9$
$9 = b$

$y = 9x + 9$

Exponential: $\frac{\Delta y}{\Delta x} = \frac{27}{18} = 1.5$

$y = a(1.5)^x$
$\frac{18}{1.5} = \frac{a(1.5)^1}{1.5}$
$12 = a$

$y = 12(1.5)^x$

Express each number in standard notation.

1. 3.65×10^5
365,000

2. 7.02×10^{-4}
0.000702

3. 8.003×10^8
800,300,000

4. 7.451×10^6
7,451,000

5. 5.91×10^0
5.91

6. 7.99×10^{-1}
0.799

7. 8.9354×10^{10}
89,354,000,000

8. 8.1×10^{-9}
0.0000000081

9. 4×10^{15}
4,000,000,000,000,000

Express each number in scientific notation.

10. 0.0000456
4.56×10^{-5}

11. 0.00001
1×10^{-5}

12. 590,000,000
5.9×10^8

13. 0.00000000012
1.2×10^{-10}

14. 0.000080436
8.0436×10^{-5}

15. 0.03621
3.621×10^{-2}

16. 433×10^4
4.33×10^6

17. 0.0042×10^{-3}
4.2×10^{-6}

18. 50,000,000,000
5×10^{10}

17. 56×10^7
5.6×10^8

18. 4740×10^5
4.74×10^8

19. 0.076×10^{-3}
7.6×10^{-5}

20. 0.0057×10^3
5.7×10^0