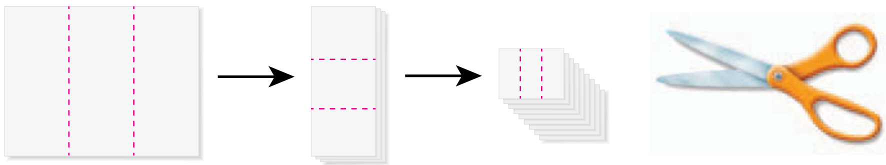
Applications

1. Cut a sheet of paper into thirds. Stack the three pieces and cut the stack into thirds. Stack all of the pieces and cut the stack into thirds again.



a. Copy and complete this table to show the number of ballots after repeating this process five times.

b. Suppose you continued this process. How many ballots would you have after 10 cuts? How many would you have after n cuts?

c. How many cuts would it take to make at least one million ballots?

Cutting Ballots

Cutting Processes	Number of Ballots
1	3
2	■
3	■
4	■
5	■

2. Chen, Lisa, Gabriel, and Artie each take a large piece of paper to make ballots. First, they cut the original piece of paper in half. Then, they cut each of those new pieces in half. Finally, they cut all of those pieces in half for a total of three cuts. They want to know how many ballots they will have without counting them. Each has a different conjecture. Who do you agree with? Explain.

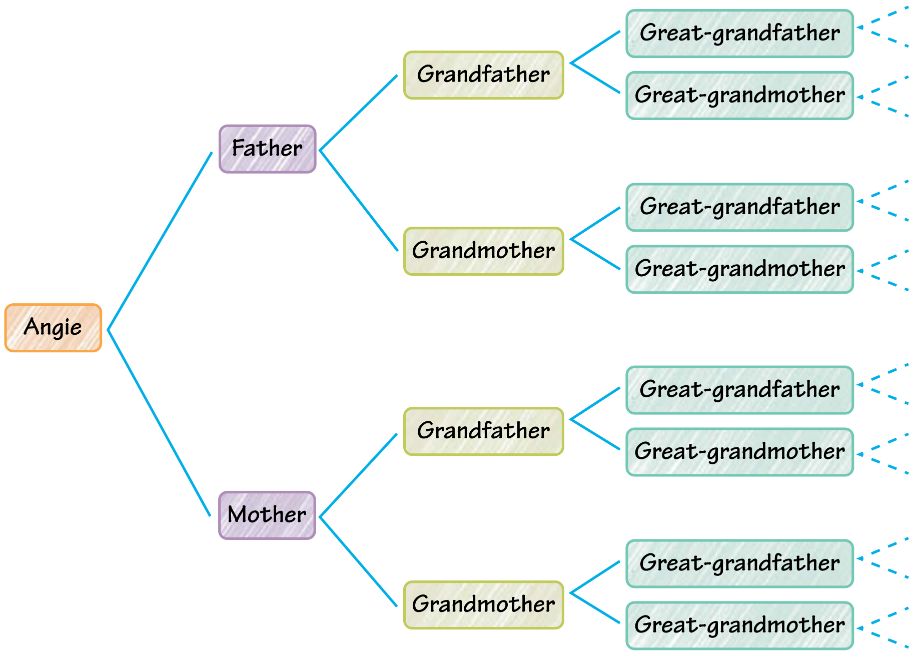
Chen's Conjecture The total number of ballots will be 2^{12} because as a group we made twelve total cuts.

Lisa's Conjecture The total number will be 8^4 because each person will have eight ballots, and there are four of us.

Gabriel's Conjecture The total number will be 4×2^3 because each person makes 2^3 ballots and there are four of us.

Artie's Conjecture The total number can't be determined using a formula. You will have to count them piece by piece.

3. Angie is studying her family's history. She discovers records of ancestors 12 generations back. She wonders how many ancestors she has from the past 12 generations. She starts to make a diagram to help her figure this out. The diagram soon becomes very complex.



a. Make a table and a graph showing the number of ancestors in each of the 12 generations.

b. Write an equation for the number of ancestors a in a given generation n.

c. What is the total number of ancestors in all 12 generations?

4. Sarah was working on Problem 1.2. She found that there will be 2,147,483,648 rubas on square 32.

   a. How many rubas will be on square 33? How many will be on square 34? How many will be on square 35?

   b. Which square would have the number of rubas shown here?

   $$2{,}147{,}483{,}648 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

   c. Use your calculator to do the multiplication in part (b). Do you notice anything strange about the answer your calculator gives? Explain.

   d. Write $2{,}147{,}483{,}648 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ in scientific notation.

   e. Write the numbers 2^{10}, 2^{20}, 2^{30}, 2^{40}, and 2^{50} in scientific notation.

   f. Explain how to write a large number in scientific notation.

For Exercises 5–7, write each number in scientific notation.

5. 100,000,000

6. 29,678,900,522

7. 11,950,500,000,000

For Exercises 8–10, write each number in standard form.

8. 6.43999001×10^8

9. 8.89234×10^5

10. $3.4348567000 \times 10^{10}$

11. What is the largest whole-number value of n that your calculator will display in standard notation?

    a. 3^n

    b. π^n

    c. 12^n

    d. 237^n

12. What is the smallest value of n that your calculator will display in scientific notation?

a. 10^n

b. 100^n

c. $1000n^n$

13. Many single-celled organisms reproduce by dividing into two identical cells.

Suppose an amoeba (uh MEE buh) splits into two amoebas every half hour.

a. A biologist starts an experiment with one amoeba. Make a table showing the number of amoebas she would have at the end of each hour over an 8-hour period.

b. Write an equation for the number of amoebas a after t hours. Which variable is the independent variable? Dependent variable?

c. How many hours will it take for the number of amoebas to reach one million?

d. Make a graph of the data (*time, amoebas*) from part (a).

e. What similarities do you notice in the pattern of change for the number of amoebas and the patterns of change for other situations in this Investigation? What differences do you notice?

14. Zak's uncle wants to donate money to Zak's school. He suggests three possible plans. Look for a pattern in each plan.

Plan 1 He will continue the pattern in this table until day 12.

School Donations

Day	1	2	3	4
Donation	$1	$2	$4	$8

Plan 2 He will continue the pattern in this table until day 10.

School Donations

Day	1	2	3	4
Donation	$1	$3	$9	$27

Plan 3 He will continue the pattern in this table until day 7.

School Donations

Day	1	2	3	4
Donation	$1	$4	$16	$64

a. Copy and extend each table to show how much money the school would receive each day.

b. For each plan, write an equation for the relationship between the day number n and the number of dollars donated d.

c. Are any of the relationships in Plans 1, 2, or 3 exponential functions? Explain.

d. Which plan would give the school the greatest total amount of money?

15. Carmelita is planning to swim in a charity swim-a-thon. Several relatives said they would sponsor her.

I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $5 if you swim 3 laps, $7 if you swim 4 laps, and so on.—**Grandmother**

I will give you $1 if you swim 1 lap, $3 if you swim 2 laps, $9 if you swim 3 laps, $27 if you swim 4 laps, and so on.—**Father**

I will give you $2 if you swim 1 lap, $3.50 if you swim 2 laps, $5 if you swim 3 laps, $6.50 if you swim 4 laps, and so on.—**Aunt Josie**

I will give you $1 if you swim 1 lap, $2 if you swim 2 laps, $4 if you swim 3 laps, $8 if you swim 4 laps, and so on.—**Uncle Sebastian**

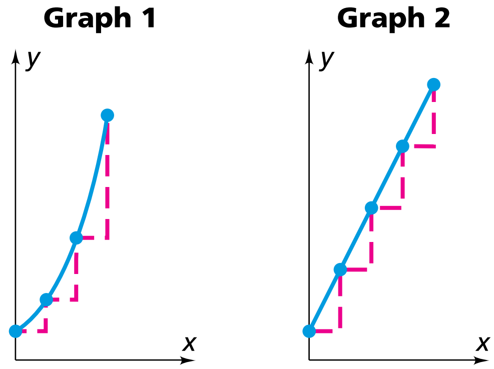
WOW! Thanks everyone for your support!—**Carmelita**

a. Decide whether each donation pattern is an *exponential function, linear function,* or *neither*.

b. For each relative, write an equation for the total donation d if Carmelita swims n laps. Which variable is the independent variable? Dependent variable?

c. For each plan, tell how much money Carmelita will raise if she swims 20 laps.

16. The graphs below represent the equations $y = 2^x$ and $y = 2x + 1$.



a. Tell which equation each graph represents. Explain your reasoning.

b. The dashed segments show the vertical and horizontal change between points at equal x intervals. For each graph, compare the vertical and horizontal change between pairs of points. What do you notice?

c. Does either equation represent an exponential function? A linear function? Explain.

For Exercises 17–21, study the pattern in each table.

a. Tell whether the relationship between x and y is a *linear function, exponential function,* or *neither*. Explain your reasoning.

b. If the relationship is a linear or exponential, give its equation.

17.

x	0	1	2	3	4	5
y	10	12.5	15	17.5	20	22.5

18.

x	0	1	2	3	4
y	1	6	36	216	1,296

19.

x	0	1	2	3	4	5	6	7	8
y	1	5	3	7	5	8	6	10	8

20.

x	0	1	2	3	4	5	6	7	8
y	2	4	8	16	32	64	128	256	512

21.

x	0	1	2	3	4	5
y	0	1	4	9	16	25

Connections

For Exercises 22–24, write each expression in exponential form.

22. $2 \times 2 \times 2 \times 2$

23. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

24. $2.5 \times 2.5 \times 2.5 \times 2.5 \times 2.5$

For Exercises 25–27, write each expression in standard form.

25. 2^{10} **26.** 10^2 **27.** 3^9

28. You know that $5^2 = 25$. How can you use this fact to evaluate 5^4?

29. The standard form for 5^{10} is 9,765,625. How can you use this fact to evaluate 5^{11}?

30. Multiple Choice Which expression is equal to one million?

A. 10^6 **B.** 6^{10} **C.** 100^2 **D.** 2^{100}

31. Use exponents to write an expression for one billion (1,000,000,000).

For Exercises 32–34, decide whether each number is more or less than one million *without using a calculator* or multiplying. Explain how you found your answer. Use a calculator to check your answer.

32. 9^6 **33.** 3^{10} **34.** 11^6

For Exercises 35–40, write the number in exponential form using 2, 3, 4, or 5 for the base.

35. 125 **36.** 64 **37.** 81

38. 3,125 **39.** 1,024 **40.** 4,096

41. Refer to Problem 1.1. Suppose 250 sheets of paper is 1 inch high.

a. How high would the stack of ballots be after 20 cuts? After 30 cuts?

b. How many cuts would it take to make a stack 1 foot high?

c. The average distance from Earth to the moon is about 240,000 miles. Which (if any) of the stacks in part (a) would reach the moon?

42. In Problem 1.2, suppose a Montarek ruba has the value of a modern U.S. penny. What are the dollar values of the rubas on squares 10, 20, 30, 40, 50, and 60?

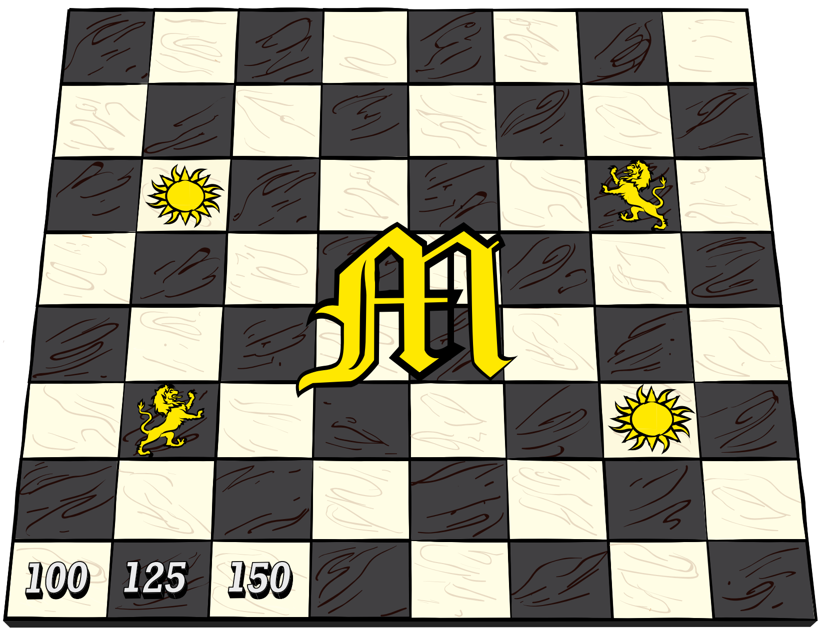
43. A ruba has the same thickness as a modern U.S. penny (about 0.06 inch). Suppose the king had been able to reward the peasant by using Plan 1 (doubling the number of rubas in each square). What would be the height of the stack of rubas on square 64?

44. One of the king's advisors suggested another plan. Put 100 rubas on the first square of a chessboard, 125 on the second square, 150 on the third square, and so on, increasing the number of rubas by 25 for each square.

a. Write an equation for the numbers of rubas r on square n. Explain the meanings of the numbers and variables in your equation.

b. Describe the graph of this plan.

c. What is the total number of rubas on the first 10 squares? The first 20 squares?



For Exercises 45–47, find the slope and *y*-intercept of the graph of each equation.

45. $y = 3x - 10$

46. $y = 1.5 - 5.6x$

47. $y = 15 + \frac{2}{5}x$

48. Write an equation whose line is less steep than the line represented by $y = 15 + \frac{2}{5}x$.

Extensions

49. Consider the two equations below.

Equation 1	**Equation 2**
$r = 3^n - 1$	$r = 3^{n-1}$

a. For each equation, find r when n is 2.

b. For each equation, find r when n is 10.

c. Explain why the equations give different values of r for the same value of n.

d. Do either of these equations represent an exponential function? Explain why.

50. The table below represents the number of ballots made by repeatedly cutting a sheet of paper in half.

Cutting Ballots

Number of Cuts	Number of Ballots
1	2
2	4
3	8
4	16

a. Write an equation for the pattern in the table.

b. Use your equation and the table to determine the value of 2^0.

c. What do you think b^0 should equal for any number b? For example, do you think 6^0 and 23^0 should equal? Explain.

51. The king tried to figure out the total number of rubas the peasant would receive under Plan 1. He noticed an interesting pattern.

a. Extend and complete this table for the first 10 squares.

Reward Plan 1

Square	Number of Rubas on Square	Total Number of Rubas
1	1	1
2	2	3
3	4	7
4	■	■

b. Describe the pattern of growth in the total number of rubas as the number of the square increases. Do either of these relationships represent an exponential function? Explain.

c. Write an equation for the relationship between the number of the square n and the total number of rubas on the board t.

d. When the total number of rubas reaches 1,000,000, how many squares will have rubas?

e. Suppose the king had been able to give the peasant the reward she requested. How many rubas would she have received?

52. Refer to Plans 1–4 in Problem 1.3.

a. Which plan should the king choose? Explain.

b. Which plan should the peasant choose? Explain.

c. Write an ending to the story of the king and the peasant.