

8-1 Study Guide and Intervention

Multiplying Monomials

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n , $a^m \cdot a^n = a^{m+n}$.
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Example 1 Simplify $(3x^6)(5x^2)$.

$$\begin{aligned} (3x^6)(5x^2) &= (3)(5)(x^6 \cdot x^2) && \text{Associative Property} \\ &= (3 \cdot 5)(x^{6+2}) && \text{Product of Powers} \\ &= 15x^8 && \text{Simplify.} \end{aligned}$$

The product is $15x^8$.

Example 2 Simplify $(-4a^3b)(3a^2b^5)$.

$$\begin{aligned} (-4a^3b)(3a^2b^5) &= (-4)(3)(a^3 \cdot a^2)(b \cdot b^5) \\ &= -12(a^{3+2})(b^{1+5}) \\ &= -12a^5b^6 \end{aligned}$$

The product is $-12a^5b^6$.

Exercises

Simplify.

1. $y(y^5)$

2. $n^2 \cdot n^7$

3. $(-7x^2)(x^4)$

4. $x(x^2)(x^4)$

5. $m \cdot m^5$

6. $(-x^3)(-x^4)$

7. $(2a^2)(8a)$

8. $(rs)(rs^3)(s^2)$

9. $(x^2y)(4xy^3)$

10. $\frac{1}{3}(2a^3b)(6b^3)$

11. $(-4x^3)(-5x^7)$

12. $(-3j^2k^4)(2jk^6)$

13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$

14. $(-5xy)(4x^2)(y^4)$

15. $(10x^3yz^2)(-2xy^5z)$

8-1 Study Guide and Intervention *(continued)***Multiplying Monomials**

Powers of Monomials An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n , $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n , $(ab)^m = a^m b^m$.

Example**Simplify $(-2ab^2)^3(a^2)^4$.**

$$\begin{aligned}
 (-2ab^2)^3(a^2)^4 &= (-2ab^2)^3(a^8) && \text{Power of a Power} \\
 &= (-2)^3(a^3)(b^2)^3(a^8) && \text{Power of a Product} \\
 &= (-2)^3(a^3)(a^8)(b^2)^3 && \text{Commutative Property} \\
 &= (-2)^3(a^{11})(b^2)^3 && \text{Product of Powers} \\
 &= -8a^{11}b^6 && \text{Power of a Power}
 \end{aligned}$$

The product is $-8a^{11}b^6$.**Exercises****Simplify.**

1. $(y^5)^2$

2. $(n^7)^4$

3. $(x^2)^5(x^3)$

4. $-3(ab^4)^3$

5. $(-3ab^4)^3$

6. $(4x^2b)^3$

7. $(4a^2)^2(b^3)$

8. $(4x)^2(b^3)$

9. $(x^2y^4)^5$

10. $(2a^3b^2)(b^3)^2$

11. $(-4xy)^3(-2x^2)^3$

12. $(-3j^2k^3)^2(2j^2k)^3$

13. $(25a^2b)^3\left(\frac{1}{5}abc\right)^2$

14. $(2xy)^2(-3x^2)(4y^4)$

15. $(2x^3y^2z^2)^3(x^2z)^4$

16. $(-2n^6y^5)(-6n^3y^2)(ny)^3$

17. $(-3a^3n^4)(-3a^3n)^4$

18. $-3(2x)^4(4x^5y)^2$

8-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a , $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b , $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 Simplify $\frac{a^4b^7}{ab^2}$. Assume neither a nor b is equal to zero.

$$\begin{aligned} \frac{a^4b^7}{ab^2} &= \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right) && \text{Group powers with the same base.} \\ &= (a^{4-1})(b^{7-2}) && \text{Quotient of Powers} \\ &= a^3b^5 && \text{Simplify.} \end{aligned}$$

The quotient is a^3b^5 .

Example 2 Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$.

Assume that b is not equal to zero.

$$\begin{aligned} \left(\frac{2a^3b^5}{3b^2}\right)^3 &= \frac{(2a^3b^5)^3}{(3b^2)^3} && \text{Power of a Quotient} \\ &= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3} && \text{Power of a Product} \\ &= \frac{8a^9b^{15}}{27b^6} && \text{Power of a Power} \\ &= \frac{8a^9b^9}{27} && \text{Quotient of Powers} \end{aligned}$$

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{5^5}{5^2}$

2. $\frac{m^6}{m^4}$

3. $\frac{p^5n^4}{p^2n}$

4. $\frac{a^2}{a}$

5. $\frac{x^5y^3}{x^5y^2}$

6. $\frac{-2y^7}{14y^5}$

7. $\frac{xy^6}{y^4x}$

8. $\left(\frac{2a^2b}{a}\right)^3$

9. $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$

10. $\left(\frac{2v^5w^3}{v^4w^3}\right)^4$

11. $\left(\frac{3r^6s^3}{2r^5s}\right)^4$

12. $\frac{r^7s^7t^2}{s^3r^3t^2}$

$$11) \frac{2n^2}{n}$$

$$12) \frac{8x^3}{10x^5}$$

$$13) \frac{12x^3}{9y^8}$$

$$14) \frac{14x^4y^7}{6x^5y^4}$$

$$15) \frac{11u^4}{17u^7v^9}$$

$$16) \frac{4y^4}{14yx^8}$$

$$17) \frac{12yx^4}{10yx^8}$$

$$18) \frac{18x^8y^8}{10x^3}$$

$$19) \frac{5n^8}{20n^8}$$

$$20) \frac{16yx^4}{9x^8y^2}$$

8-2 Study Guide and Intervention *(continued)*

Dividing Monomials

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a , $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n , $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.

$$\begin{aligned} \frac{4a^{-3}b^6}{16a^2b^6c^{-5}} &= \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right) && \text{Group powers with the same base.} \\ &= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5) && \text{Quotient of Powers and Negative Exponent Properties} \\ &= \frac{1}{4}a^{-5}b^0c^5 && \text{Simplify.} \\ &= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5 && \text{Negative Exponent and Zero Exponent Properties} \\ &= \frac{c^5}{4a^5} && \text{Simplify.} \end{aligned}$$

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{2^2}{2^{-3}}$

2. $\frac{m}{m^{-4}}$

3. $\frac{p^{-8}}{p^3}$

4. $\frac{b^{-4}}{b^{-5}}$

5. $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$

6. $\frac{(a^2b^3)^2}{(ab)^{-2}}$

7. $\frac{x^4y^0}{x^{-2}}$

8. $\frac{(6a^{-1}b)^2}{(b^2)^4}$

9. $\frac{(3st)^2u^{-4}}{s^{-1}t^2u^7}$

10. $\frac{s^{-3}t^{-5}}{(s^2t^3)^{-1}}$

11. $\left(\frac{4m^2n^2}{8m^{-1}l}\right)^0$

12. $\frac{(-2mn^2)^{-3}}{4m^{-6}n^4}$

8-2 Skills Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{6^5}{6^4}$

2. $\frac{9^{12}}{9^8}$

3. $\frac{x^4}{x^2}$

4. $\frac{r^3s^2}{r^3s^4}$

5. $\frac{m}{m^3}$

6. $\frac{9d^7}{3d^6}$

7. $\frac{12n^5}{36n}$

8. $\frac{w^4u^3}{w^4u}$

9. $\frac{a^3b^5}{ab^2}$

10. $\frac{m^7n^2}{m^3n^2}$

11. $\frac{-21w^5u^2}{7w^4u^5}$

12. $\frac{32x^3y^2z^5}{-8xyz^2}$

13. $\left(\frac{4p^7}{7s^2}\right)^2$

14. 4^{-4}

15. 8^{-2}

16. $\left(\frac{5}{3}\right)^{-2}$

17. $\left(\frac{9}{11}\right)^{-1}$

18. $\frac{h^3}{h^{-6}}$

19. $k^0(k^4)(k^{-6})$

20. $k^{-1}(\ell^{-6})(m^3)$

21. $\frac{f^{-7}}{f^4}$

22. $\left(\frac{16p^5q^2}{2p^3q^3}\right)^0$

23. $\frac{f^{-5}g^4}{h^{-2}}$

24. $\frac{15x^6y^{-9}}{5xy^{-11}}$

25. $\frac{-15w^0u^{-1}}{5u^3}$

26. $\frac{48x^6y^7z^5}{-6xy^5z^6}$