Investigation 1

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 1.1
Core 1–4
Other *Applications* 5–7; *Connections* 31

Problem 1.2
Core 10–11, 15–21, 39–42
Other *Applications* 8, 9, 12–14, *Connections* 32, 33, 43–46; unassigned choices from previous problems

Problem 1.3
Core 22, 23, 47
Other *Connections* 34; *Extensions* 48, 49; unassigned choices from previous problems

Problem 1.4
Core 25–30
Other *Applications* 24, *Connections* 35–38; *Extensions* 50; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 22 and other ACE exercises, see the CMP *Special Needs Handbook*
Connecting to Prior Units 34–38: *Moving Straight Ahead*

Applications

1. a.

Number of Cuts	Ballots
1	3
2	9
3	27
4	81
5	243

b. $3^{10} = 59{,}049$; 3^n

c. 13. After 13 cuts, there would be $1{,}594{,}323 = 3^{13}$ ballots, which is over 1 million ballots, but 3^{12} is less than 1 million.

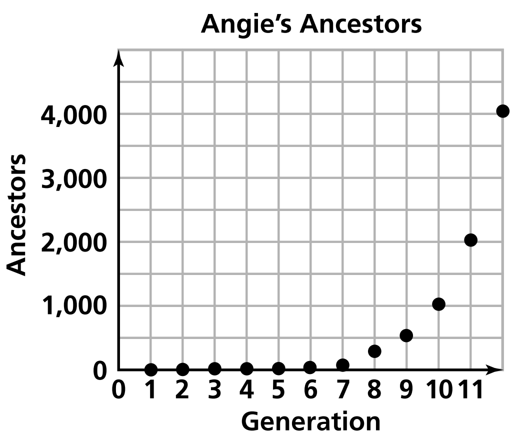
Note to the Teacher Students should be familiar with exponential notation from the grade 6 unit *Prime Time*.

2. 2^4
3. 10^7
4. $(2.5)^5$
5. 1,024
6. 100
7. 19,683
8. Because 5^2 means $5 \cdot 5$ and 5^4 means $5 \cdot 5 \cdot 5 \cdot 5$, 5^4 also equals $5^2 \cdot 5^2 = 25 \cdot 25 = 625$.
9. Because 5^{11} has one more factor of 5 than 5^{10} has, it equals $5^{10} \cdot 5 = 9{,}765{,}625 \cdot 5 = 48{,}828{,}125$.
10. A
11. 10^9
12. 9^6 is less than 1 million; possible explanation: The product of six 9s must be less than the product of six 10s, which is 10^6, or 1 million.
13. 3^{10} is less than 1 million; possible explanation: $3^{10} = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^5$. So $3^{10} = 9^5$, which is less than 10^6 (1 million).
14. 11^6 is greater than 10^6 or 1 million, because to find 11^6, you multiply 11 by itself six times. The result must be greater than if you multiply 10 by itself 6 times.
15. $5^3 = 125$
16. 2^6 or 4^3
17. 3^4
18. 5^5
19. 2^{10} or 4^5
20. 2^{12} or 4^6

21. a.

Angie's Ancestors

Generation	Ancestors
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
11	2,048
12	4,096



b. $a = 2^n$, where a is the number of ancestors and n is the generation number.

c. 8,190. You can find this by adding $2 + 4 + 8 + \ldots + 4{,}096 = 8{,}190$.

Note to the Teacher: See the page 16 for a description of how to use a calculator to find the sum of a sequence.

The ancestor pattern is identical to the pattern in the paper-cutting activity of Problem 1.1 and is very similar to the chessboard pattern in Problem 1.2. The only noticeable difference is that, in Problem 1.2, $n = 0$ doesn't make sense because there is not a square 0. (The graph for the rubas problem in Problem 1.2 is a translation of the graph that applies to both Problem 1.1 and the ancestor problem.)

If you go back 40 generations, the number of ancestors exceeds the number of people who have ever lived!

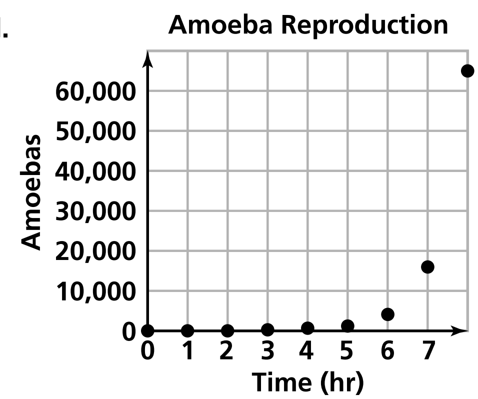
22. a. Amoeba Reproduction

Time (hr)	Amoebas
0	1
1	4
2	16
3	64
4	256
5	1,024
6	4,096
7	16,384
8	65,536

b. $a = 4^t$

c. 10 hours. At 10 hours, there will be 1,048,576 amoebas present.

d.



e. The pattern of change in the number of amoebas is similar to the pattern of change in the number of ancestors because it is exponential. The difference between the two patterns is that the number of amoebas increases more rapidly than the number of ancestors because the number of amoebas quadruples at each stage, while the number of ancestors only doubles.

23. **a.** (Figures 3, 4, and 5)

b. Plan 1: $d = 2^n$
Plan 2: $d = 3^n$
Plan 3: $d = 4^n$

c. Plan 2

24. **a.** Grandmother's and Aunt Lori's plans are linear and Uncle Jack's and Mother's plans are exponential.

b. Grandmother: $a = 2n - 1$
Mother: $a = 3^{n-1}$
Aunt Lori: $a = 1.5n + 0.50$
Uncle Jack: $a = 2^{n-1}$

c. Grandmother: $39;
Mother: $1,162,261,467;
Aunt Lori: $30.50;
Uncle Jack: $524,288

25. **a.** Graph 1 represents $y = 2^x$ because it is a curve. Graph 2 represents $y = 2x + 1$ because it is a straight line.

b. In both graphs, the horizontal change is constant. In the graph of $y = 2x + 1$, the vertical change is also constant. In the graph of $y = 2x$, the vertical change increases.

26. **a.** Linear

b. $y = 2.5x + 10$

27. **a.** Exponential

b. $y = 6^x$

28. Neither

29. **a.** Exponential

b. $y = 7(2^x)$

30. Neither (In fact, this is quadratic. Students will study quadratic relationships in the unit *Frogs, Fleas, and Painted Cubes.*)

ACE ANSWERS 1

Figure 3

Plan 1

Day	0	1	2	3	4	5	6	7	8	9	10	11	12
Donation	$1	$2	$4	$8	$16	$32	$64	$128	$256	$512	$1,024	$2,048	$4,096

Figure 4

Plan 2

Day	0	1	2	3	4	5	6	7	8	9	10
Donation	$1	$3	$9	$27	$81	$243	$729	$2,187	$6,561	$19,683	$59,049

Figure 5

Plan 3

Day	0	1	2	3	4	5	6	7
Donation	$1	$4	$16	$64	$256	$1,024	$4,096	$16,384

Connections

31. a. After 20 cuts: $1{,}048{,}576 \div 250 \approx 4{,}194$ in.; about 349.5 ft.; after 30 cuts: $1{,}073{,}741{,}824 \div 250 \approx 4{,}294{,}967$ in.; about 67.8 mi.

b. 12 cuts. A foot-high stack has $250 \times 12 = 3{,}000$ ballots. Because 11 cuts gives 2,048 ballots and 12 cuts gives 4,096 ballots, Chen would have to make at least 12 cuts to get 3,000 ballots.

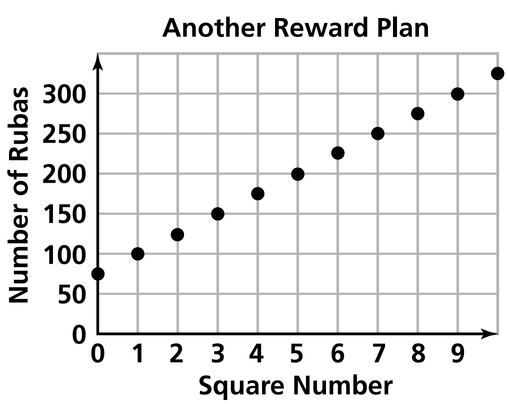
32. Square 10: \$5.12; square 20: \$5,242.88; square 30: \$5,368,709.12; square 40: \$5,497,558,138.88; square 50: about $\$5.63 \times 10^{12}$; square 60: about $\$5.76 \times 10^{15}$

33. a. When $n = 64$, the number of rubas is $2^{64-1} = 9.22 \times 10^{18}$.
The stack would have been about $(9.22 \times 10^{18}) \times 0.06 = 5.53 \times 10^{17}$ in., or 4.61×10^{16} ft, or 8.73×10^{12} mi high.

b. It is 240,000 mi $\cdot$ 5,280 ft/mi $\cdot$ 12 in./ft $\approx 1.52 \cdot 10^{10}$ in. to the moon, so it would take $1.52 \cdot 10^{10} \div 0.06 \approx 2.53 \cdot 10^{11}$ rubas to reach the moon. The stacks on squares 39 ($2.75 \cdot 10^{11}$ rubas) and above will reach the moon.

34. a. $r = 100 + 25(n - 1)$ or $r = 25n + 75$

b. The graph will look linear. You might want to ask students to draw the graph.



c. 2,125 rubas; 6,750 rubas

35. Slope: 3; y-intercept: -10

36. Slope: -5.6; y-intercept: 1.5

37. Slope: $\frac{2}{5}$; y-intercept: 15

38. Answers will vary. Any equation with a coefficient of less than $\frac{2}{5}$. Possible answers: $y = \frac{1}{5}x + 3$; $y = \frac{1}{6}x + 15$

39. a. Square 33: $2^{32} = 4{,}294{,}967{,}296$
Square 34: $2^{33} = 8{,}589{,}934{,}592$
Square 35: $2^{34} = 17{,}179{,}869{,}184$

b. Square 41. This is because it is the number of rubas on square 32 times nine more factors of 2.

c. The display probably reads 1.09951162778 E12. There is an E in the middle of the number.

d. $1.09951162778 \times 10^{12}$ (There are occasions when the calculator display will not give the last few digits exactly.)

e. $2^{10} = 1.024 \times 10^3$; $2^{20} = 1.048576 \times 10^6$;
$2^{30} = 1.073471824 \times 10^9$;
$2^{40} = 1.09951162778 \times 10^{12}$;
$2^{50} = 1.12589990684 \times 10^{15}$

f. Possible answer: To write a number in scientific notation, place a decimal in the original number to get a number greater than or equal to 1 but less than 10. To compensate for placing the decimal in the original number, multiply the new number by a power of ten that will give you back your original number.

40. 1.0×10^8

41. $2.96789005 \times 10^{10}$

42. 1.19505×10^{13}

Values given for Exercises 43–46 are for the standard screen of the TI-83. Different calculators will give different results.

43. 20

44. 20

45. 9

46. 4

Extensions

47. a. Eq. 1: $r = 3^2 - 1 = 9 - 1 = 8$
Eq. 2: $r = 3^{2-1} = 3^1 = 3$

b. Eq. 1: $r = 3^{10} - 1 = 59{,}048$
Eq. 2: $r = 3^{10-1} = 3^9 = 19{,}683$

c. The equations give different values of r because the value of n is used differently. In one equation, 1 is subtracted from n and the result becomes the exponent of 3; in the other, n is used as the exponent of 3, and 1 is subtracted from the result.

Note to the Teacher For $n \geq 0$, the result of Equation 1 will always be greater than that of Equation 2 because the exponent is greater. Subtracting from the greater exponential contribution is almost insignificant.

48. a. $b = 2^n$, where b is the number of ballots made and n is the number of cuts

b. From the table, 2^0 must equal 1. When you evaluate 2^0 with a calculator, the answer is 1.

c. The value of any number b raised to a power of 0 is 1.

Note to the Teacher Talk with students about why this makes sense. Because $b^1 = b$ and because exponents tell us how many factors of the base to use, $b \times b^0 = b^{1+0}$ must be equal to $1 + 0$ factors of b, which is just b; so b^0 should be 1 to make all of the other ideas about exponents work out. Some students might say that for the pattern to continue backwards, 2^0 must equal 1. Explain that mathematicians decided the world of mathematics would make more sense if b^0 were defined to be 1 for $b \neq 0$.

49. a. (Figure 6)

b. Each entry in the total column is 1 less than the entry in the next individual square column. Another pattern is that we can double the total rubas at a square and add 1 to get the total rubas at the next square.

c. $t = 2^n - 1$

d. The total t will exceed 1,000,000 when 20 squares have been covered: $t = 2^{20} - 1 = 1{,}048{,}575$ rubas.

e. With all 64 squares covered, the total would be $t = 2^{64} - 1 \approx 1.84 \times 10^{19}$ rubas.

50. a–c. Answers will vary.

Possible Answers to Mathematical Reflections

1. One key property of exponential growth patterns is that each time you increase the value of the independent variable by 1, you multiply the value of the dependent variable by the same constant number, the growth factor. Another key property of exponential growth patterns is that the numbers begin to grow very quickly. The graphs of exponential growth patterns are increasing curves.
2. With exponential growth patterns, you *multiply* the y-value by a constant each time the x-value increases by 1. With linear growth patterns, you *add or subtract* a constant to the y-value each time the x-value increases by 1. Also, the graphs of linear patterns are straight lines, while graphs of exponential growth patterns are curves.

Figure 6

Reward Plan 1

Square	Number of Rubas on Square	Total Number of Rubas
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255
9	256	511
10	512	1,023