ACE Applications | Connections | Extensions

Applications

1. Solve for x.

a. $(x-3)^2 - 1 = 0$

b. $(x+1)^2 - 4 = 0$

c. $(x-1)^2 - 3 = 0$

d. $(x+1)^2 - 1 = 0$

e. $4(x+1)^2 - 4 = 0$

f. $4(x-1)^2 - 3 = 0$

2. The functions below correspond to the equations you solved in Exercise 1. Use information about the vertex form and the zeroes you found in Exercise 1. Match each function to the correct graph. Explain your thinking in each case.

a. $a(x) = (x-3)^2 - 1$

b. $b(x) = (x+1)^2 - 4$

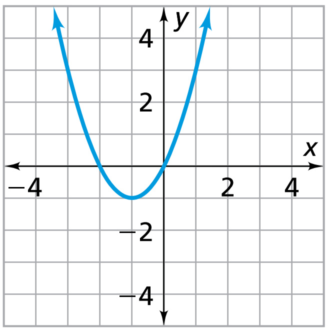
c. $c(x) = (x-1)^2 - 3$

d. $d(x) = (x+1)^2 - 1$

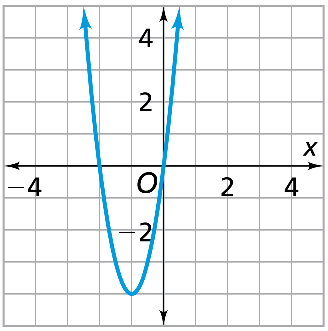
e. $e(x) = 4(x+1)^2 - 4$

f. $f(x) = 4(x-1)^2 - 3$

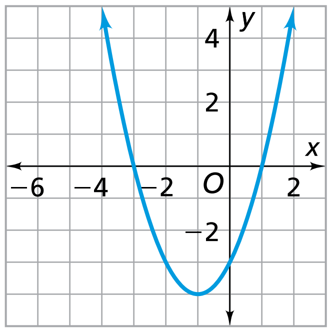
Parabola 1



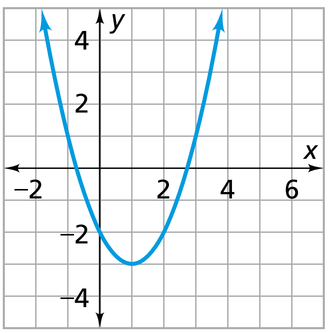
Parabola 2



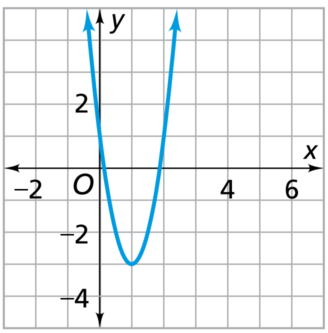
Parabola 3



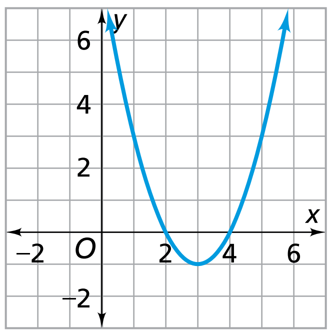
Parabola 4



Parabola 5



Parabola 6



In Exercises 3–8, expand each expression to give a perfect square trinomial.

3. $(x + 7)^2$
4. $(x - 7)^2$
5. $(2x + 7)^2$
6. $(2x - 7)^2$
7. $(x + p)^2$
8. $\left(x + \frac{q}{s}\right)^2$

For Exercises 9–14, each quadratic function is in standard form.

- **Complete the square to write each function in vertex form.**
- **Identify coordinates of the maximum or minimum point.**
- **Identify the x-intercept(s) and y-intercept.**
- **State which form is more convenient to identify coordinates of the maximum/minimum point, x-intercept(s), and y-intercept.**

9. $f(x) = x^2 + 2x - 3$
10. $g(x) = x^2 - 4x - 5$
11. $h(x) = x^2 - 6x + 5$
12. $j(x) = x^2 + 4x + 2$
13. $k(x) = -x^2 + 3x - 1$
14. $l(x) = -x^2 + 8x - 5$

For Exercises 15–20, use the Quadratic Formula to solve the equations. Give both the exact result provided by the formula and a decimal approximation, where appropriate.

15. $5x^2 + 10x - 15 = 0$
16. $3x^2 + 2x - 7 = 0$
17. $-4x^2 + 5x - 1 = 0$
18. $7x + 3x^2 = -3$
19. $0 = 2 + 3x + x^2$
20. $3x^2 - 2 = 6x$

21. Solve the equation $x^2 - 6x + 13 = 0$. Write the solutions as complex numbers in the form $a + bi$ and $a - bi$ if needed.

Write the result of the indicated sum, difference, or product in the form of a single complex number $a + bi$.

22. $(3 + 7i) + (13 - 4i)$
23. $(3 + 7i) - (13 - 4i)$
24. $(3 + 7i)(13 - 4i)$
25. $(3 + 7i)(3 - 7i)$

Connections

26. Bianca's teacher asks her students to find a quadratic function that has zeroes at $(-2, 0)$ and $(0, 0)$.

a. Bianca says that there is only one quadratic function that fits this description. She makes the following table of values for her function. Is $f(x)$ a quadratic function? Explain.

x	-2	-1	0	1	2	3
f(x)	0	-1	0	3	8	15

b. Aleshanee says she can make a different table and different graph, with the same zeroes. Is $g(x)$ a quadratic function? Explain.

x	-2	-1	0	1	2	3
g(x)	0	-4	0	12	32	60

c. How are $f(x)$ and $g(x)$ related to each other?

27. Write expressions in vertex form, $a(x - b)^2 + c$, for functions with these properties.

a. minimum point at $(3, -5)$

b. maximum point at $(1, 4)$

c. minimum point at $(-3, 1)$

d. maximum point at $(-5, -2)$

28. Write equations that have these properties.

a. solutions are whole numbers

b. solutions are integers, but not whole numbers

c. solutions are real numbers, but not rational numbers

d. solutions are complex numbers, but not real numbers

29. In each pair of calculations, write the two results in simplest possible equivalent form. Explain how the reasoning in each case is based on number system properties.

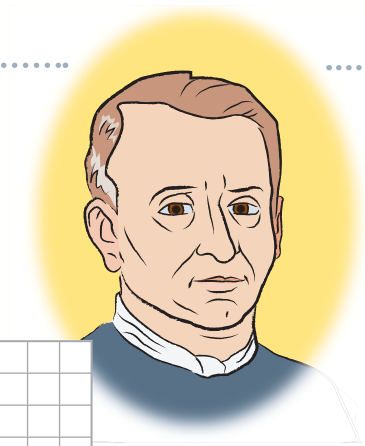
a. $(3 + 2i) + (7 - 3i)$ and $(3 + 2x) + (7 - 3x)$

b. $(3 + 2i) - (7 - 3i)$ and $(3 + 2x) - (7 - 3x)$

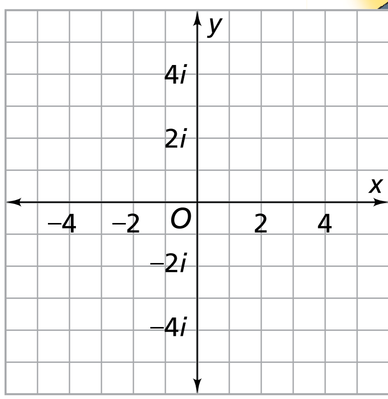
c. $(3 + 2i)(7 - 3i)$ and $(3 + 2x)(7 - 3x)$

d. $-(7 - 3i)$ and $-(7 - 3x)$

Did You Know?



In 1806, J. R. Argand developed a method for displaying complex numbers graphically. Any complex number could be displayed as a point in the coordinate plane. His method, called the *Argand Diagram*, relates the *x*-axis (real axis) with the *y*-axis (imaginary axis). The *x*-values are real numbers *a*. The *y*-values are imaginary numbers *bi*.



30. On a copy of the coordinate plane shown above, label each point with the complex number it represents.

a. $3 + 2i$ **b.** $4 - 3i$ **c.** $-2 + 4i$ **d.** $-4 - 3i$

31. Name the geometric transformations that map points representing any complex number as indicated.

a. $(3 + 2i) \rightarrow (3 - 2i)$ and, in general, $(a + bi) \rightarrow (a - bi)$

b. $(3 + 2i) \rightarrow (-3 + 2i)$ and, in general, $(a + bi) \rightarrow (-a + bi)$

c. $(3 + 2i) \rightarrow (-3 - 2i)$ and, in general, $(a + bi) \rightarrow (-a - bi)$

d. $(3 + 2i) \rightarrow (7 + 5i)$ and, in general, $(a + bi) \rightarrow (a + 4) + (b + 3)i$

Extensions

32. The following tables each show a quadratic relationship.

x	−2	−1	0	1	2	3
$f(x)$	0	−1	0	3	8	15

x	−2	−1	0	1	2	3
$g(x)$	0	−2	0	6	16	30

a. For each table, find the function that represents the table.

b. How are the functions related to each other?

33. Alejandro and Latasia are solving the equation $4x^2 + 32x + 60 = 0$. Their methods are shown below.

Alejandro's Method

$$4x^2 + 32x + 60 = 0$$
$$(2x + 10)(2x + 6) = 0$$

OR

Latasia's Method

$$4x^2 + 32x + 60 = 0$$
$$4(x^2 + 8x + 15) = 0$$
$$x^2 + 8x + 15 = 0$$
$$(x + 5)(x + 3) = 0$$

a. What are the solutions for x using Latasia's Method?

b. What are the solutions for x using Alejandro's Method?

c. Which of the two methods is correct (*Latasia's, Alejandro's, both, or neither*)?

34. Hai and Jenna were asked to use the *completing the square* strategy to solve the equation $4x^2 + 16x - 48 = 0$.

Hai's Method

First, factor 4 from the left side to get

$4(x^2 + 4x - 12) = 0$

Then, solve $x^2 + 4x - 12 = 0$ like this:

$$x^2 + 4x + \blacksquare = 12$$
$$x^2 + 4x + 4 = 16$$
$$(x + 2)^2 = 16$$
$$x + 2 = 4 \text{ or } x + 2 = -4$$
$$x = -2 \text{ or } x = -6$$

OR

Jenna's Method

$$4x^2 + 16x + \blacksquare = 48$$
$$4x^2 + 16x + 64 = 48 + 64$$
$$(2x + 8)^2 = 112$$
$$2x + 8 = \pm\sqrt{112}$$
$$2x = -8 \pm\sqrt{112}$$
$$x = \frac{-8 \pm\sqrt{112}}{2}$$

Which of these two methods is correct? Explain.

35. **a.** You can use ideas from the Quadratic Formula proof to complete the square. These ideas cover all cases where the coefficient of x^2 is a number other than 1. Explain why each step in the process below is justified.

Step 1 $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x\right) + c$

Step 2 $= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$

Step 3 $= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$

Step 4 $= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \left(c - \frac{b^2}{4a}\right)$

Step 5 $= a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

b. Follow each step in the process demonstrated above to complete the square for the expression $2x^2 + 6x + 5$.

36. Use Step 5 in the process demonstrated in Exercise 35 to write each quadratic expression in equivalent vertex form.

a. $3x^2 + 5x + 7$ **b.** $-3x^2 + 5x - 7$ **c.** $6x^2 - 5x + 4$

37. For any complex number $a + bi$, the number $a - bi$ is called its **complex conjugate**. Show that the product of a complex number and its complex conjugate is always a real number. Simplify each expression below.

a. $(3 + 2i)(3 - 2i)$ **b.** $(-3 - 2i)(-3 + 2i)$ **c.** $(a + bi)(a - bi)$

38. The relationship of each complex number to its complex conjugate can be used to define division of complex numbers.

a. Explain why each step in this calculation of $(3 + 4i) \div (1 + 2i)$ makes sense.

Step 1 $\frac{3 + 4i}{1 + 2i} = \frac{3 + 4i}{1 + 2i} \cdot \frac{1 - 2i}{1 - 2i}$

Step 2 $= \frac{(3 + 4i)(1 - 2i)}{(1 + 2i)(1 - 2i)}$

Step 3 $= \frac{11 - 2i}{5}$

Step 4 $= \frac{11}{5} - \frac{2}{5}i$

b. Write $(3 + 4i) \div (1 - 2i)$ in standard complex number form $a + bi$.

c. Write $(3 - 4i) \div (1 + 2i)$ in standard complex number form $a + bi$.

39. Xavier applied what he knew about adding fractions to problems with variables in the denominator. For example, Xavier recalled that when adding $\frac{2}{3} + \frac{1}{5}$, the common denominator is $15 = 3 \cdot 5$. He used this strategy to find the sum $\frac{2}{3} + \frac{1}{a}$.

Step 1 $\frac{2}{3} + \frac{1}{a} = \left(\frac{a}{a} \cdot \frac{2}{3}\right) + \left(\frac{3}{3} \cdot \frac{1}{a}\right)$

Step 2 $= \frac{2a}{3a} + \frac{3}{3a}$

Step 3 $= \frac{2a + 3}{3a}$

Is Xavier's method correct? Explain.

40. Use a strategy similar to Exercise 39 to write each sum as a single fraction.

a. $\frac{5}{b} + \frac{2}{5}$ **b.** $\frac{3}{d} + \frac{2}{c}$ **c.** $\frac{1}{a} + \frac{1}{4a^2}$