Applications

1. a. $x = 4$ and $x = 2$

   b. $x = 1$ and $x = -3$

   c. $x = 1 + \sqrt{3}$ and $x = 1 - \sqrt{3}$

   d. $x = 0$ and $x = -2$

   e. $x = 0$ and $x = -2$

   f. $x = 1 + \frac{\sqrt{3}}{2}$ and $x = 1 - \frac{\sqrt{3}}{2}$

2. a. $a(x) = (x - 3)^2 - 1$ is Parabola 6.

   b. $b(x) = (x + 1)^2 - 4$ is Parabola 3.

   c. $c(x) = (x - 1)^2 - 3$ is Parabola 4.

   d. $d(x) = (x + 1)^2 - 1$ is Parabola 1.

   e. $e(x) = 4(x + 1)^2 - 4$ is Parabola 2.

   f. $f(x) = 4(x - 1)^2 - 3$ is Parabola 5.

3. $(x + 7)^2 = x^2 + 14x + 49$

4. $(x - 7)^2 = x^2 - 14x + 49$

5. $(2x + 7)^2 = 4x^2 + 28x + 49$

6. $(2x - 7)^2 = 4x^2 - 28x + 49$

7. $(x + p)^2 = x^2 + 2px + p^2$

8. $\left(x + \frac{q}{s}\right)^2 = x^2 + 2\left(\frac{q}{s}\right)x + \frac{q^2}{s^2}$

9. $x^2 + 2x - 3 = (x + 1)^2 - 4$, meaning that the minimum point on the graph of $f(x)$ is $(-1, -4)$, x-intercepts are $(-3, 0)$ and $(1, 0)$, and y-intercept is $(0, -3)$.

10. $x^2 - 4x - 5 = (x - 2)^2 - 9$, meaning that the minimum point on the graph of $g(x)$ is $(2, -9)$, x-intercepts are at $(-1, 0)$ and $(5, 0)$, and y-intercept is $(0, -5)$.

11. $x^2 - 6x + 5 = (x - 3)^2 - 4$, meaning that the minimum point on the graph of $h(x)$ is $(3, -4)$, x-intercepts are $(5, 0)$ and $(1, 0)$, and y-intercept is $(0, 5)$.

12. $x^2 + 4x + 2 = (x + 2)^2 - 2$, meaning that the minimum point on the graph of $j(x)$ is $(-2, -4)$, x-intercepts are $(-2 + \sqrt{2}, 0)$ and $(-2 - \sqrt{2}, 0)$, and y-intercept is $(0, 2)$.

13. $-x^2 + 3x - 1 = -\left(x - \frac{3}{2}\right)^2 + \frac{5}{4}$, meaning that the maximum point on the graph of $k(x)$ is $\left(\frac{3}{2}, \frac{5}{4}\right)$, x-intercepts are $\left(\frac{3}{2} + \frac{\sqrt{5}}{2}, 0\right)$ and $\left(\frac{3}{2} - \frac{\sqrt{5}}{2}, 0\right)$, and y-intercept is $(0, -1)$.

14. $-x^2 + 8x - 5 = -(x - 4)^2 + 11$, meaning that the maximum point on the graph of $l(x)$ is $(4, 11)$, x-intercepts are $(4 + \sqrt{11}, 0)$ and $(4 - \sqrt{11}, 0)$, and y-intercept is $(0, -5)$.

15. $5x^2 + 10x - 15 = 0$ has solutions $x = \frac{-10}{10} \pm \frac{\sqrt{100 - (-300)}}{10}$ or $x = 1$ and -3.

16. $3x^2 + 2x - 7 = 0$ has solutions $x = \frac{-2}{6} \pm \frac{\sqrt{4 - (-84)}}{6}$ or $x \approx 1.23$ and $x \approx -1.90$.

17. $-4x^2 + 5x - 1 = 0$ has solutions $x = \frac{-5}{-8} \pm \frac{\sqrt{25 - 16}}{-8}$ or $x = 1$ and $x = 0.25$.

18. $7x + 3x^2 = -3$ has solutions $x = \frac{-7}{6} \pm \frac{\sqrt{49 - 36}}{6}$ or $x \approx -0.57$ and $x \approx -1.77$.

19. $0 = 2 + 3x + x^2$ has solutions $x = \frac{-3}{2} \pm \frac{\sqrt{9 - 8}}{2}$ or $x = -2$ and $x = -1$.

20. $3x^2 - 2 = 6x$ has solutions $x = \frac{6}{6} \pm \frac{\sqrt{36 - (-24)}}{6}$ or $x \approx 2.29$ and $x \approx -0.29$.

21. $x = 3 \pm 2i$

22. $(3 + 7i) + (13 - 4i) = 16 + 3i$

23. $(3 + 7i) - (13 - 4i) = -10 + 11i$

24. $(3 + 7i)(13 - 4i) = 67 + 79i$

25. $(3 + 7i)(3 - 7i) = 58$

Connections

26. a. The second differences appear to be constant. This is characteristic of a quadratic function.

b. The second differences appear to be constant. This is a characteristic of a quadratic function.

c. $g(x)$ appears to be a stretch of $f(x)$ by a factor of 4.

27. a. $f(x) = (x - 3)^2 - 5$ has minimum point at $(3, -5)$.

b. $f(x) = -(x - 1)^2 + 4$ has maximum point at $(1, 4)$.

c. $f(x) = (x + 3)^2 + 1$ has minimum point at $(-3, 1)$.

d. $f(x) = -(x + 5)^2 - 2$ has maximum point at $(-5, -2)$.

Note: The value of a does not necessarily have to be 1 or -1. In general, when a is positive, the vertex is a minimum point. When a is negative, the vertex is a maximum point.

28. There are many specific equations that have the requested properties. We offer here only one example of each.

a. solvable with whole numbers: $x + 5 = 8$

b. solvable with integers, but not whole numbers: $x + 8 = 5$

c. solvable with rational numbers, but not integers: $8x = 5$

d. solvable with real numbers, but not rational numbers: $x^2 = 2$

e. solvable with complex numbers, but not real numbers: $x^2 = -1$

29. We won't offer explanations in detail for each case, only note that the commutative, associative, and distributive properties and integer arithmetic are involved in every case.

a. $(3 + 2i) + (7 - 3i) = 10 - i$ and $(3 + 2x) + (7 - 3x) = 10 - x$

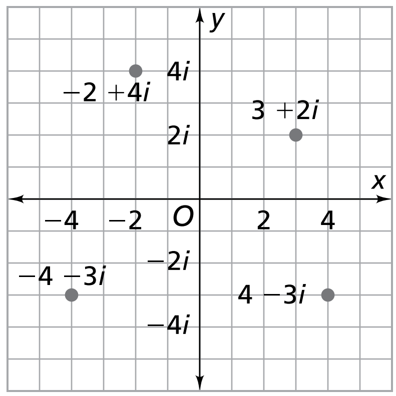
b. $(3 + 2i) - (7 - 3i) = -4 + 5i$ and $(3 + 2x) - (7 - 3x) = -4 + 5x$

c. $(3 + 2i)(7 - 3i) = 27 + 5i$ and $(3 + 2x)(7 - 3x) = 21 + 5x - 6x^2$

Note: The difference of expressions observed here is due to the fact that $i^2 = -1$.

d. $-(7 - 3i) = -7 + 3i$ and $-(7 - 3x) = -7 + 3x$

30.



31. a. $(a + bi) \rightarrow (a - bi)$ is reflection across the x-axis.

b. $(a + bi) \rightarrow (-a + bi)$ is reflection across the y-axis.

c. $(a + bi) \rightarrow (-a - bi)$ is a half-turn centered at $(0, 0)$ or $0 + 0i$.

d. $(a + bi) \rightarrow (a + 4) + (b + 3)i$ is a translation 4 units to the right and 3 units up.

Extensions

32. a. Using the symmetry around $x = -1$ in the given table for $f(x)$, the vertex of $f(x)$ is at $(-1, -1)$. The equation of a quadratic function with this vertex could be $f(x) = (x + 1)^2 - 1$. From the table, this also has zeroes at $x = -2$ and $x = 0$. So, the equation is $f(x) = (x + 1)^2 - 1$.

Again, using the symmetry around $x = -1$ in the given table for $g(x)$, the vertex of $g(x)$, is at $(-1, -2)$. The equation of a quadratic function with this vertex could be $g(x) = (x + 1)^2 - 2$. The zeros for this rule are not $(-2, 0)$ and $(0, 0)$. Starting with the zeros we have $g(x) = x(x + 2)$. This rule, however, does not have a vertex at $(-1, -2)$. We can stretch this, using a factor of 2, to keep the same zeros but a vertex of $(-1, -2)$. Therefore, $g(x) = 2x(x + 2)$ or in vertex form $g(x) = 2(x + 1)^2 - 2$.

b. $g(x) = 2f(x)$

33. a. Using Latasia's method, the solutions are $x = -5$ and $x = -3$.

b. Using Alejandro's method, the solutions are $x = -5$ and $x = -3$.

c. Both methods are correct, giving $x = -5$ and $x = -3$. Alejandro's reasoning is correct because he recognizes that the product $4(x^2 + 8x + 15)$ will be zero only when $x^2 + 8x + 15 = 0$. The fact that he didn't do anything to the right side of the equation in his first step is not a problem. He simply replaced one expression with another equivalent expression.

34. Hai's method is correct; Jenna did not factor $4x^2 + 16x + 64$ correctly. Instead of adding 64 to both sides, she should have added only 16, because the coefficient of x^2 is 4. Also, expanding $(2x + 8)^2$ would produce $4x^2 + 32x + 64$.

35. a. The reason column will be as follows.

Step 1: Factor a from the first two terms.

Step 2: Add 0 in the form $\left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$.

Step 3: Substitute $\frac{b^2}{4a^2}$ for $\left(\frac{b}{2a}\right)^2$.

Step 4: Regroup terms, noticing that $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \left(x + \frac{b}{2a}\right)^2$.

Step 5: Write the perfect square trinomial as the square of a binomial.

b.

$$\begin{aligned} 2x^2 + 6x + 5 &= 2(x^2 + 3x) + 5 \\ &= 2\left(x^2 + 3x + \left(\tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2\right) + 5 \\ &= 2\left(x^2 + 3x + \tfrac{9}{4} - \tfrac{9}{4}\right) + 5 \\ &= 2\left(x^2 + 3x + \tfrac{9}{4}\right) + \left(5 - \tfrac{18}{4}\right) \\ &= 2\left(x + \tfrac{3}{2}\right)^2 + \tfrac{1}{2} \end{aligned}$$

36. a. $3x^2 + 5x + 7 = 3\left(x + \frac{5}{6}\right)^2 + \frac{59}{12}$

b. $-3x^2 + 5x - 7 = -3\left(x - \frac{5}{6}\right)^2 - \frac{59}{12}$

c. $6x^2 - 5x + 4 = 6\left(x - \frac{5}{12}\right)^2 + \frac{71}{24}$

37. a. $(3 + 2i)(3 - 2i) = 13$

b. $(-3 - 2i)(-3 + 2i) = 13$

c. $(a + bi)(a - bi) = a^2 + b^2$

38. a. Explanations: (1) Multiply by a form of 1; (2) Multiply fractions; (3) Calculate each complex number product and simplify; (4) Use distributive property to split up fraction.

b. $(3 + 4i) \div (1 - 2i) = -1 + 2i$

c. $(3 - 4i) \div (1 + 2i) = -1 - 2i$

39. The method is correct. It uses multiplication by various forms of 1 to get the common denominator and then add numerators.

40. a. $\frac{5}{b} + \frac{2}{5} = \frac{2b + 25}{5b}$

b. $\frac{3}{d} + \frac{2}{c} = \frac{3c + 2d}{cd}$

c. $\frac{1}{a} + \frac{1}{4a^2} = \frac{4^2 + a}{4a^3}$