Applications

1. Filling rate of container 1 is shown best by Graph 2, container 2 by graph 3, and container 3 by Graph 1.

2. On both graphs, the indicated motion characteristics are shown as follows:
   - The bus is speeding up when the graph curves upward
   - The bus is slowing down when the graph curves downward
   - The bus is moving at a constant speed when the graph is a straight line segment
   - The bus is stopped when the graph is horizontal

   a. (See Figure 1.)

   b. (See Figure 2.)

3. **a.** $d(0) = 20$ says the water depth is 20 feet at midnight.

   b. $d(2) = 17.5$ says the water depth is 17.5 feet at 2 A.M.

   c. $d(4) = 12.5$ says the water depth is 12.5 feet at 4 A.M.

   d. $d(6) = 10$ says the water depth is 10 feet at 6 A.M.

   e. $d(9) = 15$ says the water depth is 15 feet at 9 A.M.

   f. $d(3) = 15$ and $d(9) = 15$ say that the water depth is 15 feet at both 3 A.M. and 9 A.M.

4. Domain is $0 \leq t \leq 12$.
   Range is $10 \leq d(t) \leq 20$.

5. **a.** $f(7) = -9$

   b. $f(-3) = 11$

   c. $f(-6) = 17$

6. **a.** $g(7) = 84$

   b. $g(-3) = -6$

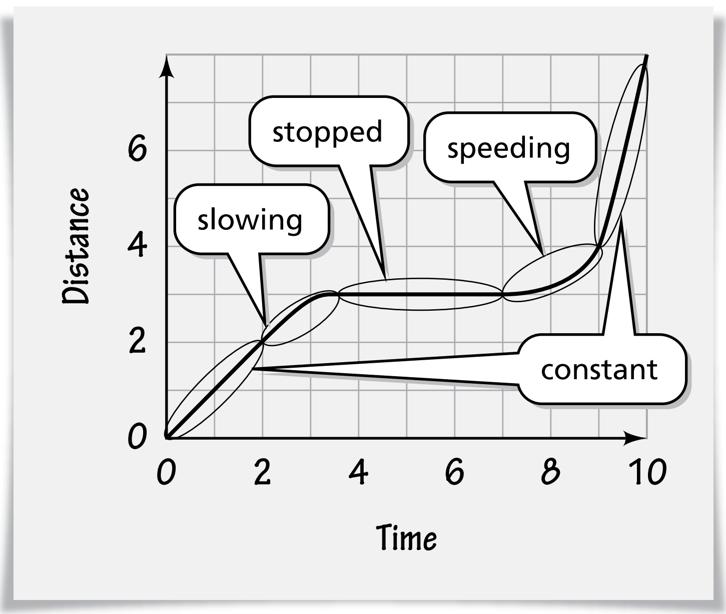
   c. $g(1 \text{ or } -6) = 6$

7. **a.** $h(2) = 1$

   b. $h(-1) = 8$

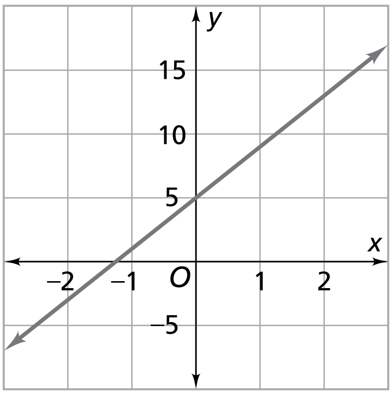
   c. $h(0) = 4$

Figure 1



8. Domain and range of $f(x) = -2x + 5$ are both all real numbers. Domain of $g(x) = x^2 + 5x$ is all real numbers, but range is all real numbers greater than or equal to -6.25 (students could discover this by graphing the function). Domain of $h(x) = 4(0.5)^x$ is all real numbers and range is all positive real numbers.

9. a. $f(x) = 4x + 5$ has domain and range all real numbers and the (partial) graph below.



b. $g(x) = x^2 + 2$ has domain all real numbers, range all real numbers greater than or equal to 2, and the (partial) graph below.

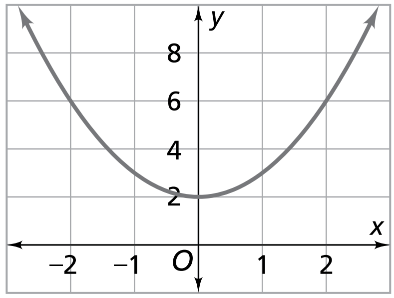
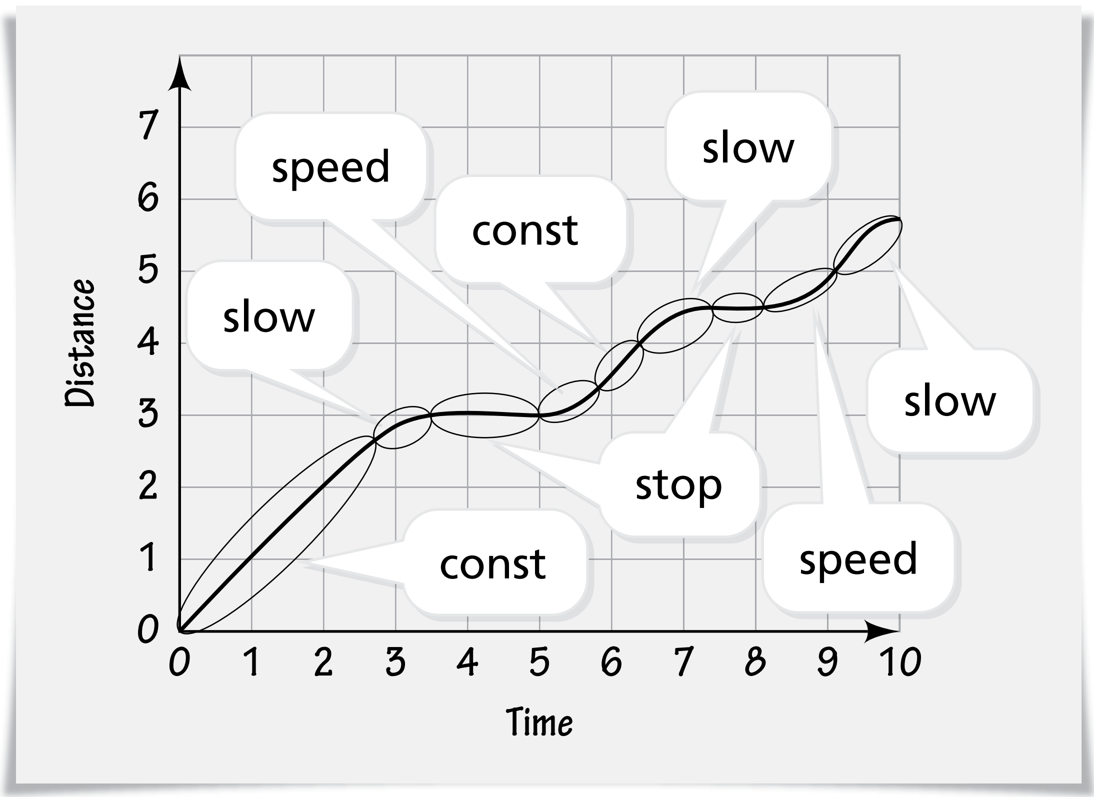
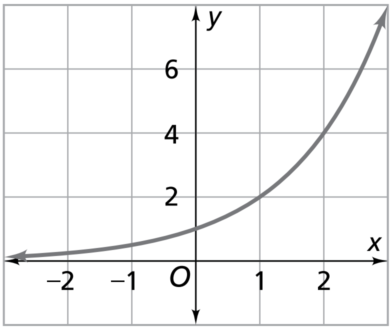


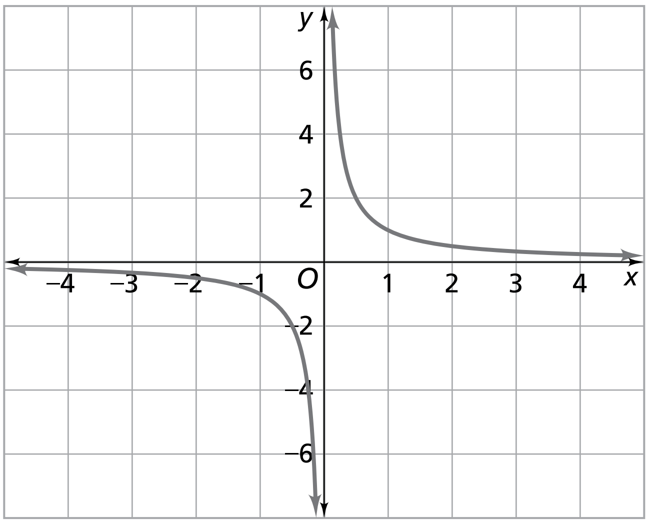
Figure 2



c. $h(x) = 2^x$ has domain all real numbers, range all positive real numbers, and the (partial) graph below.



d. $j(x) = \frac{1}{x}$ has domain and range all nonzero real numbers and the (partial) graph below.



10. a. yes

b. yes

c. no

d. yes

11. Domain and range from graphs (answers approximate)
Domain of $f(x)$ is $-4 \leq x \leq 0$, and range is $-7 \leq y \leq 1$.
Domain of $g(x)$ is $-2 \leq x \leq 5$, and range is $0.5 \leq y \leq 8$.
Domain of $h(x)$ is $-4 \leq x \leq 4$, and range is $-2 \leq y \leq 2$.

12. a. $f(-4) = -7$

b. $g(4) = 5$

c. $h(1) = -1$

d. $f(-2) = 1$

e. $g(0) = 1$

f. $h(-3) = h(0) = h(3) = 2$

13. a. $c(0.5) = 4$

b. $c(1.0) = 4$

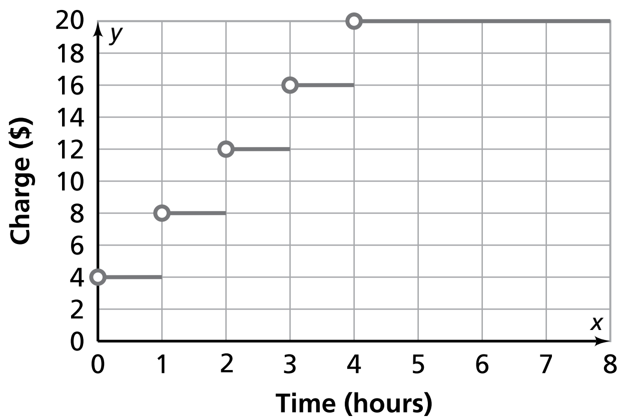
c. $c(2.5) = 12$

d $c(5.0) = 20$

e. $c(5.5) = 20$

f. $c(8.0) = 20$

g.



Note: The left endpoint of each step is not included in the graph. The right endpoint is. This means there is no time value for which there are two different charges. The conventional way to show this is to make an open circle on the end not included. Since this detail is not the main focus of this Investigation, the teacher can choose to ask students for this or not.

14. Graph C is the best representation of the situation because the function drops in steps each day, except on Saturday and Sunday.

15. a. $r(1.6) = 2$

b. $c(1.6) = 2$

c. $f(1.6) = 1$

d. $r(-1.6) = -2$

e. $c(-1.6) = -1$

f. $f(-1.6) = -2$

g. $r(-1.3) = -1$

h. $c(-1.3) = -1$

i. $f(-1.3) = -2$

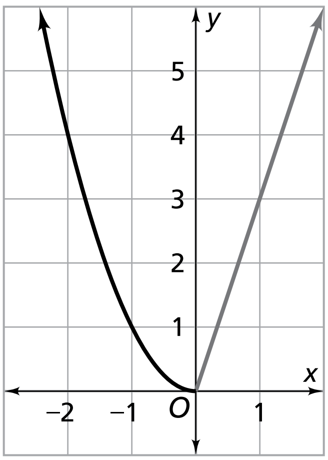
j. $r(-2.5 \le x < -1.5) = -2$

k. $c(-3 < x \le -2) = -2$

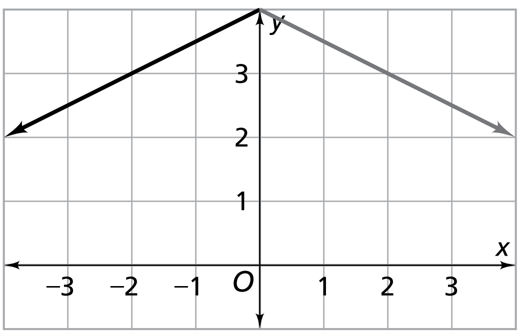
m. $f(-2 \le x < -1) = -2$

16. For the requested graphs we show only enough of the domain and range to illustrate the piecewise character of the function.

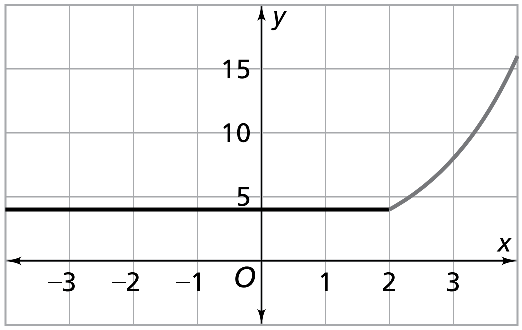
a. $y = \begin{cases} x^2 \text{ if } x \le 0 \\ 3x \text{ if } x > 0 \end{cases}$



b. $y = \begin{cases} \frac{1}{2}x + 4 \text{ if } x < 0 \\ -\frac{1}{2}x + 4 \text{ if } x \ge 0 \end{cases}$



c. $y = \begin{cases} 4 \text{ if } x < 2 \\ 2^x \text{ if } x \ge 2 \end{cases}$



17. a. The water level is rising at a rate of 1 inch per second during the first two seconds.

b. The water level is rising at a rate of $\frac{1}{3}$ inch per second during the time from 2 to 8 seconds.

18. Only Chelsea is correct, because the y-intercept of the line on the right of the diagram will not be 1.

19. a. $f^{-1}(x) = x \div 6$

b. $g^{-1}(x) = x + 4$

c. $h^{-1}(x) = \frac{(x + 4)}{6}$

d. $j^{-1}(x) = x^2$

e. $k^{-1}(x) = \sqrt{\frac{x}{4}}$ or $\frac{\sqrt{x}}{2}$

f. $m^{-1}(x) = -\frac{3}{x}$

Note: Part (f) is tricky, but if you start with $y = -\frac{3}{x}$ and solve for x in terms of y, you get $x = -\frac{3}{y}$.

20. a. Domain and range of both the function and its inverse are all real numbers.

b. Domain and range of both the function and its inverse are all real numbers.

c. Domain and range of both the function and its inverse are all real numbers.

d. Domain and range of $j(x) = \sqrt{x}$ are all nonnegative real numbers. To use $y = x^2$ as the inverse of $j(x)$, we need to restrict its domain to nonnegative numbers, though the same rule could be applied to all real numbers in other circumstances.

e. Domain and range of $k(x) = 4x^2$ are all real numbers and all nonnegative real numbers, respectively. However, since every real number and its opposite are assigned the same image by this squaring rule, there is a problem defining an inverse. We need to restrict the domain of $k(x)$ to either nonnegative numbers or nonpositive numbers to make a well-defined inverse possible. If $k(x)$ is given the domain nonpositive numbers, then the rule for its inverse would be $k^{-1}(x) = -\sqrt{\frac{x}{4}}$ because by convention the symbol $\sqrt{\frac{x}{4}}$ implies the positive square root of $\frac{x}{4}$.

f. Domain and range of $m(x) = -\frac{3}{x}$ will both be all nonzero real numbers.

Connections

21 **a.** $f(121) = 11$

b. $f\left(\frac{1}{4}\right) = \frac{1}{2}$

c. $f(m^2) = |m|$

d. $f(4q^2) = 2|q|$

22. **a.** $f(4) = 256$

b. $f(-2) = \frac{1}{16}$

c. $f(0) = 1$

d. $f\left(\frac{1}{2}\right) = 2$

e. $f(a) = 4^a$

f. $f(b + 2) = 4^{b+2}$

23. **a.** $g(3) = 9$

b. $g\left(\frac{1}{2}\right) = \frac{1}{4}$

c. $g(-d) = d^2$

d. $g\left(\frac{n}{2m}\right) = \frac{n^2}{4m^2}$

24. **a.** $j(-7) = -3.5$

b. $j(0) = 0$

c. $j(2s) = s$

d. $j\left(\frac{r}{t}\right) = \frac{r}{2t}$

25. **a.** The domain of $r(x) = \sqrt{x}$ is not all real numbers because negative numbers do not have real-number square roots (though the complex numbers to be developed in Investigation 4 provide such roots).

b. The domain of $s(x) = \frac{1}{x}$ is not all real numbers because division by 0 is not defined.

26. **a.** Your height changes continuously (although slowly).

b. The price of a one-scoop cone at your favorite ice cream shop will probably increase in increments of 5, 10, 20, or more cents over time, not continuously. So the graph will be that of a step function.

c. Your age changes continuously; however, it is common to report age only in whole years using a floor function algorithm. In that case, reported age will produce a step function graph.

d. The number of questions left to answer as you work on homework will change in whole-number increments, producing a decreasing step function over time.

27. **a.** $7 + 12 = 19$; $12 + 7 = 19$; $7 = 19 - 12$; and $12 = 19 - 7$

b. $4 \times 3 = 12$; $3 \times 4 = 12$; $3 = 12 \div 4$; and $4 = 12 \div 3$

28. Inverse operations performed in order combine to return to the starting number. For example, $3 + 7 - 7 = 3$ and $3 \times 7 \div 7 = 3$. When an inverse function is performed following an original function, the result is to return all domain elements as if nothing had happened to them. In function notation, $f^{-1}(f(x)) = x$.

29. Solve each of these linear equations using ideas about fact families and inverse operations.

a. $x + 7 = 12$ when $x = 12 - 7$, or when $x = 5$

b. $5x = 35$ when $x = 35 \div 5$, or when $x = 7$

c. $5x + 7 = 82$ when $x = \frac{(82 - 7)}{5}$, or when $x = 15$

Note: Other orders of steps are possible.

d. $\frac{7}{x} = 12$ when $7 = 12x$, or when $7 \div 12 = x$

e. $\frac{9}{(x - 2)} = 3$ when $9 = 3(x - 2)$, or $\frac{9}{3} = x - 2$, or $3 + 2 = x$, or $5 = x$

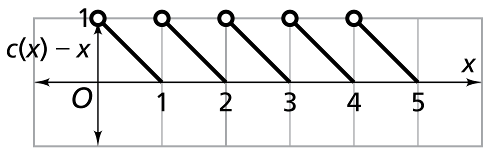
Note: Other orders of steps are possible.

f. $\frac{5}{x} + 7 = 8$ when $\frac{5}{x} = 8 - 7$, or $\frac{5}{x} = 1$, or $5 = x$

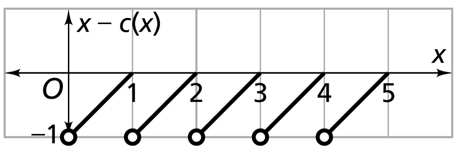
Extensions

30. a. (See Figure 3.)

b. $c(x) - x$



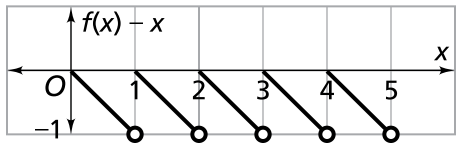
c. $x - c(x)$



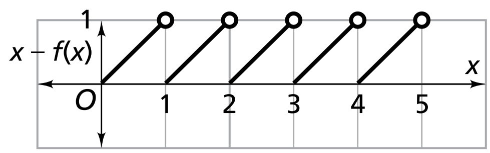
d. When you multiply a function by -1, you reflect its graph over the x-axis.

31. a. (See Figure 4.)

b. $f(x) - x$



c. $x - f(x)$



d. When you multiply a function by -1, you reflect its graph over the x-axis.

Figure 3

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
$c(x) - x$	0	0.75	0.5	0.25	0	0.75	0.5	0.25	0	0.75	0.5
$x - c(x)$	0	–0.75	–0.5	–0.25	0	–0.75	–0.5	–0.25	0	–0.75	–0.5

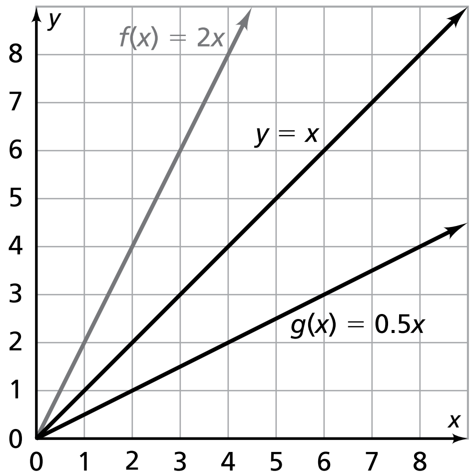
Figure 4

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
$f(x) - x$	0	–0.25	–0.5	–0.75	0	–0.25	–0.5	–0.75	0	–0.25	–0.5
$x - f(x)$	0	0.25	0.5	0.75	0	0.25	0.5	0.75	0	0.25	0.5

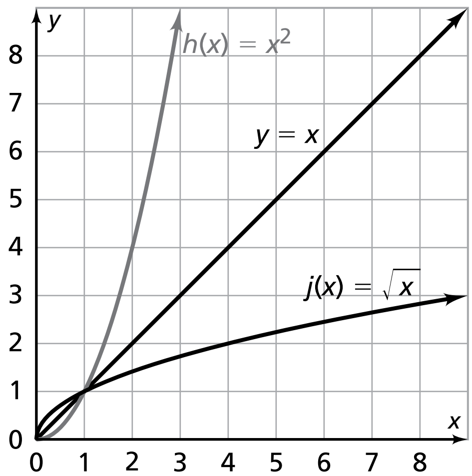
32. The ceiling function has domain all real numbers and range all integers.

33. In each pair of graphs, the function and its inverse are symmetric across the line $y = x$.

a. $f(x) = 2x$ and $g(x) = 0.5x$



b. $h(x) = x^2$ and $j(x) = \sqrt{x}$



34. a. A variety of answers are possible. Perhaps the simplest familiar examples would be $s(x) = x^2$ and $v(x) = |x|$.

b. Again, a variety of answers are possible. The examples that might come to mind most easily for students are the rounding functions discussed in Problem 1.3. The taxi-fare and time-payment examples in that same problem have domains that are sets of integers, but their domains are only positive numbers.

c. Perhaps the simplest familiar example is $r(x) = -\sqrt{x}$.

35. Margaret has correctly interpreted the notation, which says "Find g of g of 1."

36. a. Students may find the coordinates of the intersection point using a graphing calculator or other technology. They also may use the method of successive approximations, starting with the equation $5x + 35 = 5(2^x)$. This is equivalent to $x + 7 = 2^x$. Trying integer values of x is the first approximation.

$$1 + 7 = 8 > 2^1 = 2$$

$$2 + 7 = 9 > 2^2 = 4$$

$$3 + 7 = 10 > 2^3 = 8$$

$$4 + 7 = 11 < 2^4 = 16$$

So the intersection point has an x-value between 3 and 4, and a y-value between 8 and 16. The second approximation uses a noninteger value of x.

$$3.5 + 7 = 10.5 < 2^{3.5} \approx 11.3$$

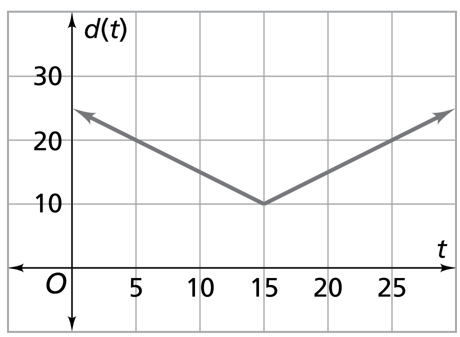
So the intersection point has an x-value between 3 and 3.5. Further approximations will give more accurate estimates of the coordinates of the intersection point.

b. The graph of $f(x)$ is plotted from the coordinates of that function. Any intersection of the graph of $f(x)$ and the graph of $g(x)$ is a point that those two functions have in common. At that point, both the x-values and the y-values of the two functions are equal. Therefore, the x-coordinate at that point is a solution of the equation $f(x) = g(x)$.

37. The required function is defined piecewise.

a. For $0 \le t \le 15$, the rule is $d(t) = 25 - t$. For $15 \le t \le 30$, the rule is $d(t) = t - 5$.

b.



c. If distance is measured as the crow flies, you use the Pythagorean Theorem to calculate it at any time t. For $0 \le t \le 15$, the rule is $d(t) = \sqrt{100 + (15 - t)^2}$.

For $15 \le t \le 30$, the rule is $d(t) = \sqrt{100 + (t - 15)^2}$.