

Real Life Situations Modeled With Quadratic Equations

Solve the following problems using what you know about key features of parabolas. All these equations can be factored. For each problem, indicate which key feature will give you your answer and **show all work** needed for calculating. (No $-b/2a$ or quadratic formula work will be accepted.)

- The height h in feet of a projectile launched vertically upward from the top of a 32-foot tall bridge is given by $h = 32 + 16t - 16t^2$, where t is time in seconds.
 - When does the projectile reach a maximum height?
 - How long will it take the projectile to strike the ground?

$$\begin{aligned} -16t^2 + 16t + 32 &= 0 \\ -16(t^2 - t - 2) &= 0 \\ -16(t-2)(t+1) &= 0 \end{aligned}$$

$$\text{LO9: } \frac{2-1}{2} = 0.5$$

$$\begin{array}{r} \swarrow \\ t-2=0 \\ +2 \quad +2 \\ \hline t=2 \end{array} \quad \begin{array}{r} \searrow \\ t+1=0 \\ -1 \quad -1 \\ \hline t=-1 \end{array}$$

- It will reach its maximum height after **0.5 seconds**.
- It will take **2 seconds** after the launch to hit the ground.

- The height h in feet of a projectile launched vertically upward from the top of a bridge is given by $h = 280 + 72t - 16t^2$ where t is time in seconds.

- How high is the top of the bridge? **280 ft.**
- When does the projectile reach a maximum height? **2.25 sec**
- How long will it take the projectile to strike the ground (not the top of the bridge)? **7 sec**

$$\begin{aligned} h &= -16t^2 + 72t + 280 \\ &= -8(2t^2 - 9t - 35) \\ &= -8(2t^2 - 14t + 5t - 35) \\ &= -8(2t(t-7) + 5(t-7)) \\ &= -8(t-7)(2t+5) \end{aligned}$$

$$\begin{array}{r} \swarrow \\ t-7=0 \\ t=7 \end{array} \quad \begin{array}{r} \searrow \\ 2t+5=0 \\ \frac{2t}{2} = \frac{-5}{2} \\ t = -2.5 \end{array}$$

$$\begin{aligned} \text{LOS} &= \frac{7-2.5}{2} \\ &= 2.25 \end{aligned}$$

3. A company's weekly revenue in dollars is given by $R = 2000x - 2x^2$ where x is the number of items produced during a week.

a. What number of items will produce the maximum revenue? **500 items**

b. What will the maximum total revenue be? **\$500,000**



a. $R = 2000x - 2x^2$

$R = 2x(1000 - x)$

$\downarrow$
 $0 = 2x$
 $0 = x$

$\downarrow$
 $1000 - x = 0$
 $\quad \quad \quad +x \quad +x$
 $\quad \quad \quad \hline x = 1000$

$LOS = \frac{0 + 1000}{2}$
 $= 500$

b. $R = 2x(1000 - x)$
 $= 2(500)(1000 - 500)$
 $= 500,000$

4. The formula below gives the height of an object thrown from a building 160 feet high with an initial speed of 48 ft/sec: $h = -16t^2 + 48t + 160$, where t is measured in seconds.

a. Find the time it takes for the object to hit the ground. **5 sec.**

b. Find the maximum height of the object. **196 feet**

a. $-16(t^2 - 3t + 10)$
 $-16(t - 5)(t + 2)$

$\downarrow$
 $t - 5 = 0$
 $\quad +5 \quad +5$
 $\quad \quad \hline t = 5$

$\downarrow$
 $t + 2 = 0$
 $\quad -2 \quad -2$
 $\quad \quad \hline t = -2$

$\nearrow$
5 seconds to
hit the ground

b. $LOS = \frac{5 - 2}{2} = 1.5$

$h = -16(1.5 - 5)(1.5 + 2)$
 $h = 196 \text{ ft.}$

5. The height h in feet of a projectile launched vertically upward from the top of a 96-foot tall tower when time t is measured in seconds, is given by $h = 96 + 80t - 16t^2$.

- a. How long will it take the projectile to strike the ground? **6 sec.**
 b. What is the maximum height that the projectile reaches? **196 ft.**

$$\begin{aligned} \text{a. } h &= -16(t^2 - 5t - 6) \\ &= -16(t - 6)(t + 1) \end{aligned}$$

$\swarrow$ $\searrow$
 $t = 6$ $t = -1$

$$\begin{aligned} \text{b. } \text{LDS} &= \frac{6 - 1}{2} = 2.5 \\ h &= -16(2.5 - 6)(2.5 + 1) \\ h &= 196 \text{ ft.} \end{aligned}$$

6. A model rocket is projected straight upward from the ground level according to the height equation $h = -16t^2 + 192t$, where h is the height in feet and t is the time in seconds.

- a. At what time is the height of the rocket maximum? **6 sec.**
 b. What is the maximum height? **576 ft.**



$$\begin{aligned} \text{a. } h &= -16t^2 + 192t \\ &= -16t(t - 12) \end{aligned}$$

$\swarrow$ $\searrow$
 $-16t = 0$ $t - 12 = 0$
 $t = 0$ $\frac{+12 \quad +12}{t = 12}$

$$\begin{aligned} \text{b. } h &= -16t(t - 12) \\ &= -16(6)(6 - 12) \\ &= 576 \text{ ft.} \end{aligned}$$

$$\begin{aligned} \text{LDS} &= \frac{12 + 0}{2} \\ &= 6 \text{ sec.} \end{aligned}$$

7. A relief package is released from a helicopter at 1600 feet. The height of the package can be modeled by the equation $h = -16t^2 + 1600$, where h is the height of the package in feet and t is the time in seconds.

- a. What does the number 1600 in the equation represent? **1600 feet is how high the package is above the ground when it is dropped.**
 b. How long it will take for the package to hit the ground?

$$\begin{aligned}
 h &= -16t^2 + 1600 \\
 &= -16(t^2 - 100) \\
 &= -16(t+10)(t-10)
 \end{aligned}$$

$$\begin{array}{r}
 0 = t + 10 \\
 -10 \quad -10 \\
 \hline
 -10 = t
 \end{array}$$

$$\begin{array}{r}
 0 = t - 10 \\
 +10 \quad +10 \\
 \hline
 10 = t
 \end{array}$$

It will take **10 sec** for the package to hit the ground.

8. The height of a flare fired from the deck of a ship in distress can be modeled by $h = -16t^2 + 104t + 56$, where h is the height of the flare above water and t is the time in seconds.

- a. How high above the surface of the water is the deck of the ship? **The deck is 56 feet above the water.**
 b. How long will the flare be in the air? **7 sec.**

$$\begin{aligned}
 h &= -16t^2 + 104t + 56 \\
 &= -8(2t^2 - 13t - 7) \\
 &= -8(2t^2 - 14t + t - 7) \\
 &= -8(2t(t-7) + 1(t-7)) \\
 &= -8(2t+1)(t-7)
 \end{aligned}$$

$$\begin{array}{r}
 2t + 1 = 0 \\
 -1 \quad -1 \\
 \hline
 2t = -1 \\
 \frac{2t}{2} = \frac{-1}{2} \\
 t = -0.5
 \end{array}$$

$$\begin{array}{r}
 t - 7 = 0 \\
 +7 \quad +7 \\
 \hline
 t = 7
 \end{array}$$

The flare is in the air for **7 sec.**

9. During a game of golf, Kayley hits her ball out of a sand trap. The height of the golf ball is modeled by the equation $h = -16t^2 + 20t - 4$, where h is the height in feet and t is the time in seconds since the ball was hit.

- a. What does the -4 in the equation tell you about the situation?

The base of the sand trap is 4 feet below ground level.

- b. Find how long it takes Kayley's golf ball to hit the ground.

$$\begin{aligned} & -16t^2 + 20t - 4 \\ & -4(4t^2 - 5t + 1) \\ & -4(4t^2 - 4t - t + 1) \\ & -4(4t(t-1) - 1(t-1)) \\ & -4(4t-1)(t-1) \end{aligned}$$

It will take 1 second for the ball to hit the ground.

$$\begin{array}{l} \swarrow \\ 4t-1=0 \\ \begin{array}{r} +1 \quad +1 \\ \hline 4t=1 \\ \frac{4}{4} \quad \frac{1}{4} \\ t=0.25 \end{array} \end{array}$$

$$\begin{array}{l} \searrow \\ t-1=0 \\ \begin{array}{r} +1 \quad +1 \\ \hline t=1 \end{array} \end{array}$$

- c. How high was Kayley able to hit the golf ball? (This is tricky, think about the situation.)

$$\begin{aligned} \text{LOS} &= \frac{0.25+1}{2} \\ &= 0.625 \end{aligned}$$

$$\begin{aligned} h &= -4(4(0.625)-1)(0.625-1) \\ &= 2.25 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{Total Height} &= 2.25 + 4 \\ &= 6.25 \text{ ft} \end{aligned}$$

- d. How long after Kayley hit the ball will it reach its highest point?

The ball will reach its highest point in 0.625 sec.

10. A pilot is flying at approximately 10,000 feet and is forced to eject from her jet. The following equation models the height, H , (given in feet), of the pilot over time, t , (given in seconds), after she is ejected from the jet and parachutes to the ground.

$$H = -16t^2 + 624t + 4320$$

- What is the maximum height (relative to the ground) reached by the pilot after being ejected?
- How many feet above the jet was the pilot ejected?
- How long did it take before the pilot landed on the ground from her highest point?



$$h = -16t^2 + 624t + 4320$$

$$= -16(t^2 - 39t - 270)$$

$$= -16(t^2 - 45t + 6t - 270)$$

$$= -16(t(t-45) + 6(t-45))$$

$$= -16(t+6)(t-45)$$

$$\begin{array}{r} \swarrow \\ t+6=0 \\ -6 \quad -6 \\ \hline t=-6 \end{array}$$

$$\begin{array}{r} \searrow \\ t-45=0 \\ +45 \quad +45 \\ \hline t=45 \end{array}$$

$$LOS = \frac{45-6}{2} = 19.5$$

a. Maximum Height:

$$\begin{aligned} h &= -16(19.5+6)(19.5-45) \\ &= 10,404 \text{ ft} \end{aligned}$$

b. $10,404 - 10,000 = 404 \text{ ft.}$

she was ejected 404 ft. above the plane.

c. $45 - 19.5 = 25.5 \text{ sec.}$

↑
Landing
time

↑
highest
point
time