Applications

1. A signal flare is fired into the air from a boat. The height h of the flare in feet after t seconds is $h = -16t^2 + 160t$.

   a. How high will the flare travel? When will it reach this maximum height?

   b. When will the flare hit the water?

   c. Explain how you could use a table and a graph to answer the questions in parts (a) and (b).

2. A model rocket is launched from the top of a hill. The table shows how the rocket's height above ground level changes as it travels through the air.

   a. How high above ground level does the rocket travel? When does it reach this maximum height?

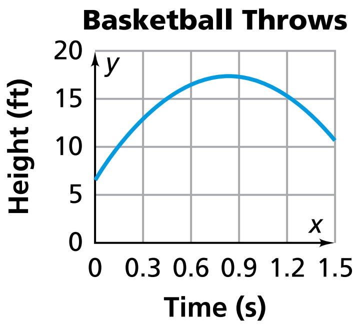
   b. From what height is the rocket launched?

   c. How long does it take the rocket to return to the top of the hill?

Height of Model Rocket

Time (seconds)	Height (feet)
0.00	84
0.25	99
0.50	112
0.75	123
1.00	132
1.25	139
1.50	144
1.75	147
2.00	148
2.25	147
2.50	144
2.75	139
3.00	132
3.25	123
3.50	112
3.75	99
4.00	84

3. A basketball player throws the ball, attempting to make a basket. The graph shows the height of the ball starting when it leaves the player's hands.



a. Estimate the height of the ball when the player releases it.

b. When does the ball reach its maximum height? What is the maximum height?

c. How long does it take the ball to reach the basket (a height of 10 feet)?

4. The highest dive in the Olympic Games is from a 10-meter platform. The height h in meters of a diver t seconds after leaving the platform can be estimated by the equation $h = 10 + 4.9t - 4.9t^2$.

a. Make a table of the relationship between time and height.

b. Sketch a graph of the relationship between time and height.

c. When will the diver hit the water's surface? How can you find this answer by using your graph? How can you find this answer by using your table?

d. When will the diver be 5 meters above the water?

e. When is the diver falling at the fastest rate? How is this shown in the table? How is this shown in the graph?

5. Kelsey jumps from a diving board, springing up into the air and then dropping feet-first. The distance d in feet from her feet to the pool's surface t seconds after she jumps is $d = -16t^2 + 18t + 10$.

a. What is the maximum height of Kelsey's feet during this jump? When does the maximum height occur?

b. When do Kelsey's feet hit the water?

c. What does the constant term 10 in the equation tell you about Kelsey's jump?

6. The equation $h = -16t^2 + 48t + 8$ describes how the height h of a ball in feet changes over time t.

a. What is the maximum height reached by the ball? Explain how you could use a table and a graph to find the answer.

b. When does the ball hit the ground? Explain how you could use a table and a graph to find the answer.

c. Describe the pattern of change in the height of the ball over time. Explain how this pattern would appear in a table and a graph.

d. What does the constant term 8 mean in this context?

For Exercises 7–10, complete parts (a)–(d).

a. Sketch a graph of the equation.

b. Find the x- and y-intercepts. Label these points on your graph.

c. Draw and label the line of symmetry.

d. Label the coordinates of the maximum or minimum point.

7. $y = 9 - x^2$

8. $y = 2x^2 - 4x$

9. $y = 6x - x^2$

10. $y = x^2 + 6x + 8$

11. **a.** How can you tell from a quadratic equation whether the graph will have a maximum point or a minimum point?

b. How are the x- and y-intercepts of the graph of a quadratic function related to its equation?

c. How are the x- and y-intercepts related to the line of symmetry?

For Exercises 12–17, predict the shape of the graph of the equation. Give the maximum or minimum point, the x-intercepts, and the line of symmetry. Use a graphing calculator to check your predictions.

12. $y = x^2$

13. $y = -x^2$

14. $y = x^2 + 1$

15. $y = x^2 + 6x + 9$

16. $y = x^2 - 2$

17. $y = x(4 - x)$

18. A cube with edges of length 12 centimeters is built from centimeter cubes. The faces of the large cube are painted. How many of the centimeter cubes will have

a. three painted faces?
b. two painted faces?
c. one painted face?
d. no painted faces?

19. Four large cubes are built from centimeter cubes. The faces of each large cube are painted. In parts (a)–(d), determine the size of the large cube. Then, tell how many of its centimeter cubes have 0, 1, 2, and 3 painted faces.

a. For Cube A, 1,000 of the centimeter cubes have no painted faces.

b. For Cube B, 864 of the centimeter cubes have one painted face.

c. For Cube C, 132 of the centimeter cubes have two painted faces.

d. For Cube D, 8 of the centimeter cubes have three painted faces.

20. a. Copy and complete each table. Describe the pattern of change.

x	x
1	■
2	■
3	■
4	■
5	■

x	x^2
1	■
2	■
3	■
4	■
5	■

x	x^3
1	■
2	■
3	■
4	■
5	■

b. For each table, tell which column in the painted-cubes table in Problem 4.3 has a similar pattern. Explain.

21. Consider the functions described by these equations. Are any of them similar to functions in the painted-cubes situation? Explain.

$y_1 = 2(x - 1)$ $\quad$ $y_2 = (x - 1)^3$ $\quad$ $y_3 = 4(x - 1)^2$

For Exercises 22–25, match the equation with its graph. Then, give the line of symmetry for each graph and explain how to locate it.

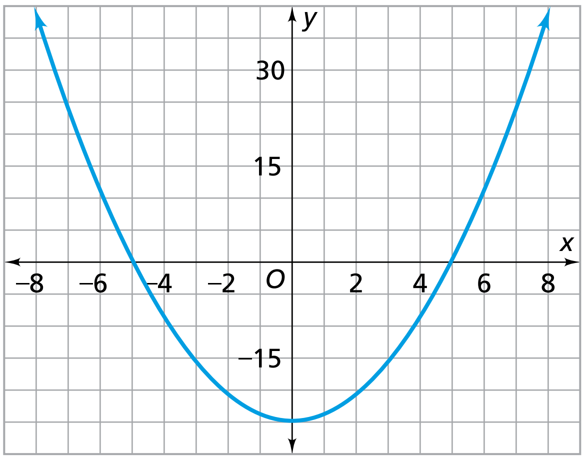
22. $y = (x + 7)(x + 2)$

23. $y = x(x + 3)$

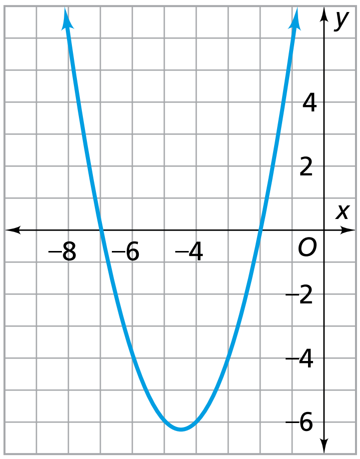
24. $y = (x - 4)(x + 6)$

25. $y = (x - 5)(x + 5)$

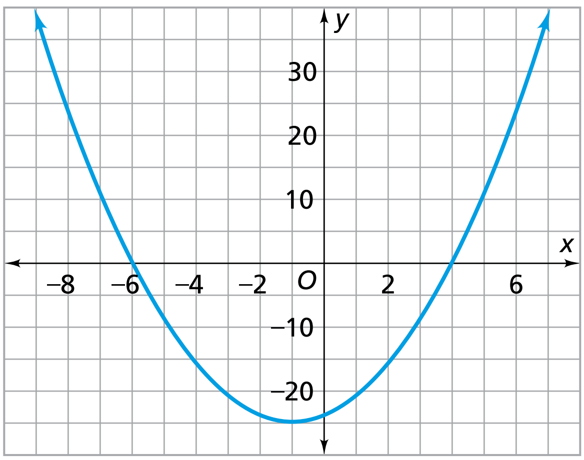
Graph A



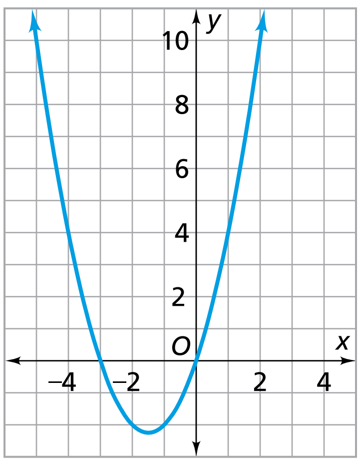
Graph B



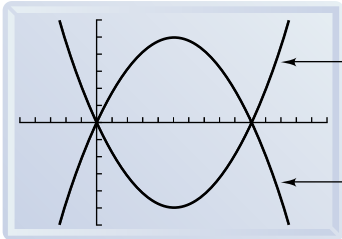
Graph C



Graph D



26. **a.** How are the graphs at the right similar?



b. How are the graphs different?

c. The maximum value for $y = x(10 - x)$ occurs when $x = 5$. How can you find the y-coordinate of the maximum value?

d. The minimum value for $y = x(x - 10)$ occurs when $x = 5$. How can you find the y-coordinate of the minimum value?

27. **Multiple Choice** Which quadratic equation has x-intercepts at $(3, 0)$ and $(-1, 0)$?

A. $y = x^2 - 1x + 3$ **B.** $y = x^2 - 2x + 3$ **C.** $y = 3x^2 - 1x$ **D.** $y = x^2 - 2x - 3$

Connections

28. **a.** Describe the patterns of change in each table. (Look closely. You may find more than one.) Explain how you can use the patterns to find the missing entry.

Table 1

x	y
0	25
1	50
2	100
3	200
4	400
5	■

Table 2

x	y
–3	3
–2	6
–1	9
0	12
1	15
2	■

Table 3

x	y
2	6
3	12
4	20
5	30
6	42
7	■

Table 4

x	y
–2	21
–1	24
0	25
1	24
2	21
3	■

b. Tell which equation matches each table.

$y_1 = x^2 - 12$ $y_2 = x(x + 1)$ $y_3 = 25 - x^2$

$y_4 = (x)(x)(x)$ $y_5 = 3(x + 4)$ $y_6 = 25(2)^x$

c. Which tables represent quadratic functions? Explain.

d. Do any of the tables include the maximum y-value for the relationship?

e. Do any of the tables include the minimum y-value for the relationship?

29. A potter wants to increase her profits by changing the price of a particular style of vase. Using past sales data, she writes these two equations relating income I to selling price p:

$$I = (100 - p)p \text{ and } I = 100p - p^2$$

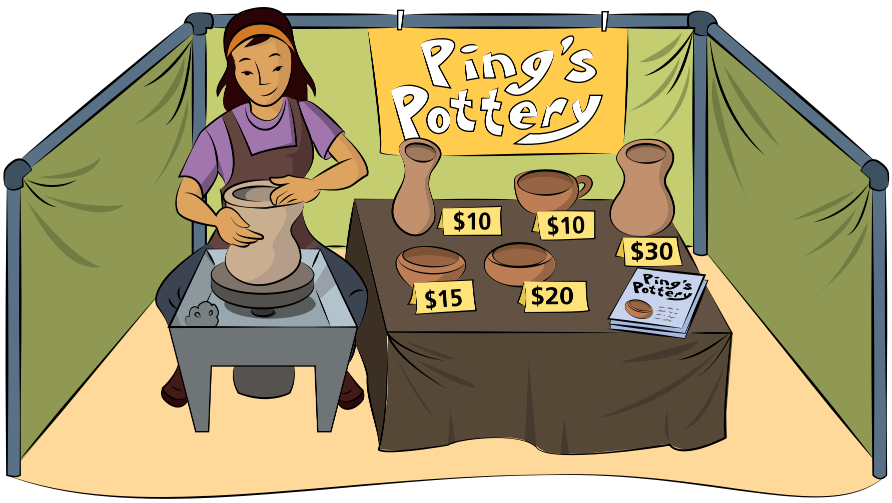
a. Are the two equations equivalent? Explain.

b. Show that $I = 100 - p^2$ is not equivalent to the original equations.

c. It costs \$350 to rent a booth at a craft fair. The potter's profit for the fair will be her income minus the cost of the booth. Write an equation for the profit M as a function of the price p.

d. What price would give the maximum profit? What will the maximum profit be?

e. For what prices will there be a profit rather than a loss?



30. Eggs are often sold by the dozen. When farmers send eggs to supermarkets, they often stack the eggs in bigger containers that are 12 eggs long, 12 eggs wide, and 12 eggs high.

a. How many eggs are in each layer of the container?

b. How many eggs are there in an entire container?

31. A square has sides of length x.

a. Write formulas for the area A and perimeter P of the square in terms of x.

b. Suppose the side lengths of the square are doubled. How do the area and perimeter change?

c. How do the area and perimeter change if the side lengths are tripled?

d. What is the perimeter of a square if its area is 36 square meters?

e. Make a table of side length, perimeter, and area values for squares with whole-number side lengths from 0 to 12.

f. Sketch graphs of the data (*side length, area*) and (*side length, perimeter*) from your table.

g. Tell whether the patterns of change in the tables and graphs suggest linear, quadratic, or exponential functions, or none of these. Explain.

32. A cube has edges of length x.

a. Write a formula for the volume V of the cube in terms of x.

b. Suppose the edge lengths of the cube double. How does the volume change?

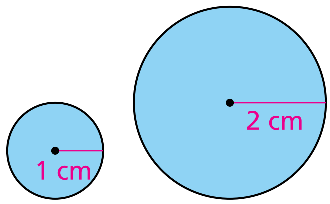
c. How does the surface area and volume change if the edge lengths triple?

d. Make a table for cubes with whole-number edge lengths from 0 to 12. Title the columns “Side Length,” “Surface Area,” and “Volume.”

e. Sketch graphs of the data (*edge length, surface area*) and (*edge length, volume*) from your table.

f. Tell whether the patterns of change in the tables and graphs suggest linear, quadratic, or exponential functions, or none of these.

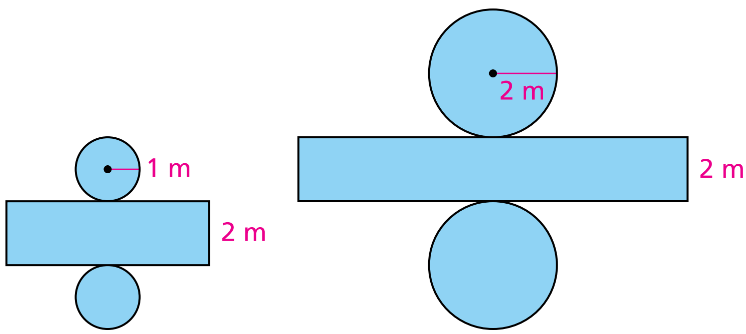
33. **a.** Find the areas of these circles.



b. Copy and complete this table. Is the relationship between the area and the radius quadratic? Explain.

Radius (cm)	1	2	3	4	x
Area (cm^2)	■	■	■	■	■

c. Below are nets for two cylinders with heights of 2 meters. Find the surface areas of the cylinders.



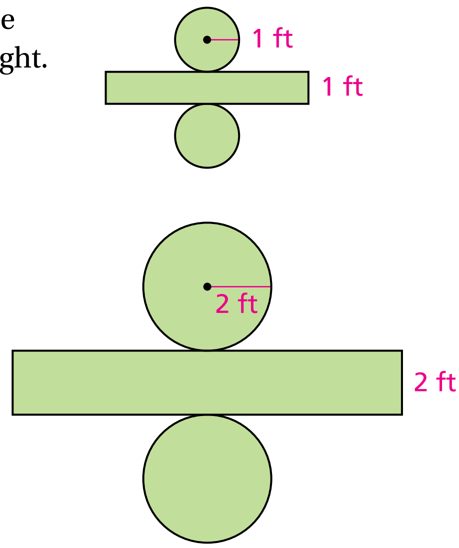
d. Copy and complete this table. Is the relationship between the surface area and the radius quadratic? Explain.

Radius (m)	1	2	3	4	x
Height (m)	2	2	2	2	2
Surface Area (m^2)	■	■	■	■	■

34. **Multiple Choice** The equation $h = 4 + 63t - 16t^2$ represents the height h of a baseball in feet t seconds after it is hit. After how many seconds will the ball hit the ground?

A. 2 seconds **B.** 4 seconds **C.** 5 seconds **D.** 15 seconds

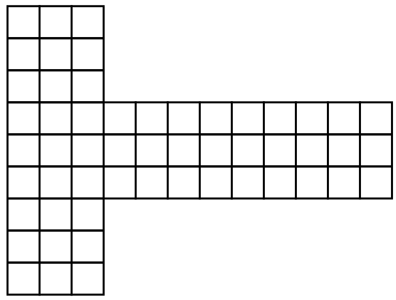
35. **a.** Copy and complete the table to show surface areas of cylinders with equal radius and height. Use the nets shown.



Radius (ft)	1	2	3	4	x
Height (ft)	1	2	3	4	x
Surface Area (ft^2)	■	■	■	■	■

b. Is the relationship between surface area and radius a quadratic function? Explain.

36. At the right is a net of a cube, divided into square units.



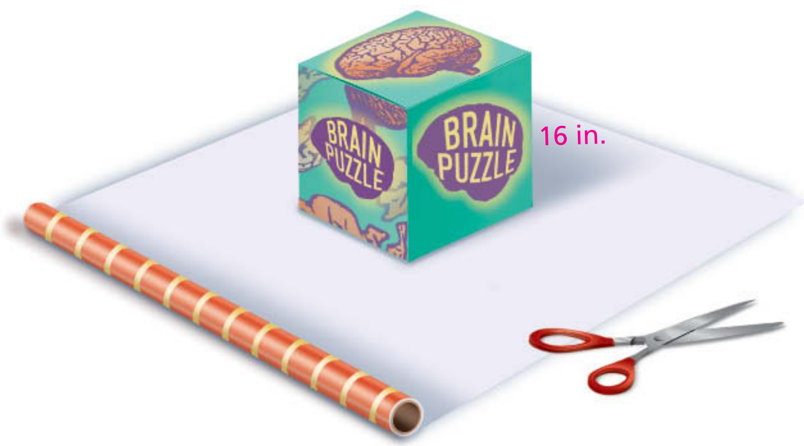
a. What is the edge length of the cube?

b. Find the surface area and volume of the cube.

c. Draw a net for a cube with a volume of 64 cubic units. What is the length of each edge of the cube? What is the surface area of the cube?

d. What formula relates the edge length of a cube to its volume? Is this relationship a quadratic function? Explain.

37. Silvio wants to gift wrap the cubic box shown. He has 10 square feet of wrapping paper. Is this enough to wrap the gift? Explain.



38. Multiple Choice Which table could represent a quadratic function?

F.

x	y
–3	–3
–2	–2
–1	–1
0	0
1	1
2	2
3	3

G.

x	y
–3	1
–2	2
–1	3
0	4
1	3
2	2
3	1

H.

x	y
1	0
2	2
3	6
4	12
5	20
6	30
7	42

J.

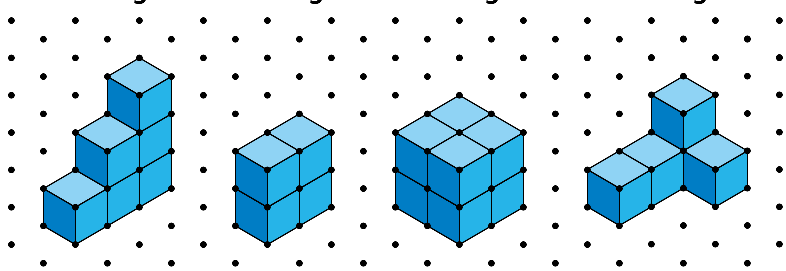
x	y
–1	10
0	7
1	4
2	1
3	4
4	7
5	10

39. Multiple Choice Suppose $y = x^2 - 4x$ and $y = 0$. What are all the possible values for x?

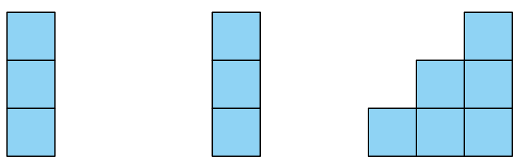
A. -4 **B.** 0 **C.** 4 or 0 **D.** -4 or 0

40. The cube buildings below are shown from the front right corner.

Building 1 **Building 2** **Building 3** **Building 4**

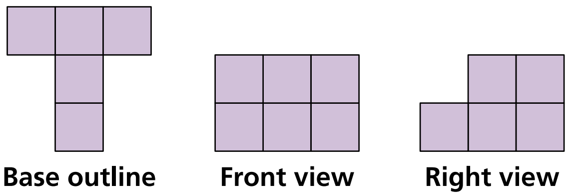


These drawings show the base outline, front view, and right view of Building 1. Draw these views for the other three buildings.

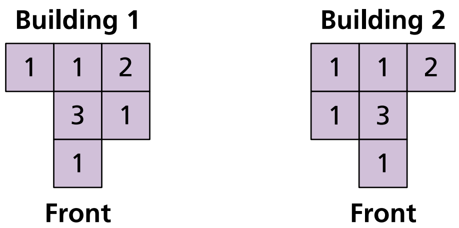


Base outline **Front view** **Right view**

41. Below are three views of a cube building. Draw a building that has all three views and has the greatest number of cubes possible. You may want to use isometric dot paper.



42. Below are base plans for cube buildings. A *base plan* shows the shape of the building's base and the number of cubes in each stack.



Make a drawing of each building from the front right corner. You may want to use isometric dot paper.

For Exercises 43–46, evaluate the expression for the given values of *x*.

43. $x(x - 5)$ for $x = 5$ and $x = -5$

44. $3x^2 - x$ for $x = 1$ and $x = \frac{1}{3}$

45. $x^2 + 5x + 4$ for $x = 2$ and $x = -4$

46. $(x - 7)(x + 2)$ for $x = -2$ and $x = 2$

47. Match the equations, graphs, and properties. Each equation is given in factored form. The window of the graphs is shown at the right.

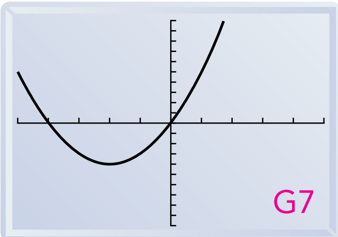
WINDOW FORMAT
Xmin=−5
Xmax=5
Xscl=1
Ymin=−10
Ymax=10
Yscl=1
Xres=1

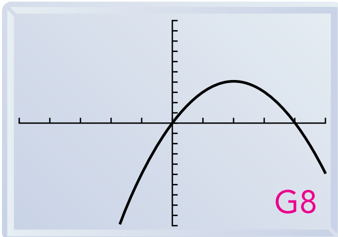
Equations:

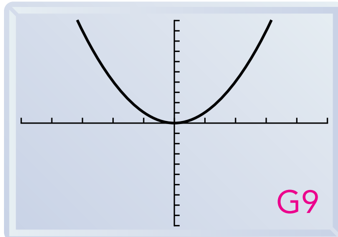
$y_1 = x^2$ $\qquad$ $y_2 = x(x - 4)$

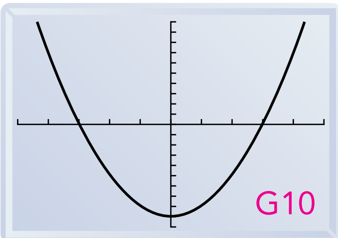
$y_3 = (x + 3)(x - 3)$ $\qquad$ $y_4 = (x + 3)(x + 3)$

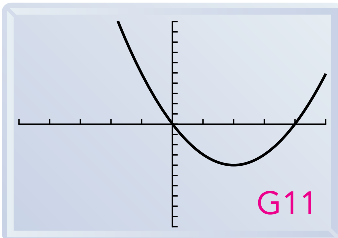
$y_5 = x(4 - x)$ $\qquad$ $y_6 = x(x + 4)$

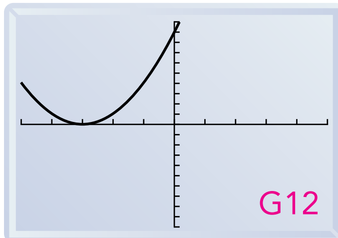












Function Properties
- Two x-intercepts
- y-intercept $= -9$
- Line of symmetry: $x = 0$

P13

Function Properties
- Two x-intercepts
- y-intercept $= 0$
- Line of symmetry: $x = -2$

P14

Function Properties
- One x-intercept
- y-intercept $= 0$
- Line of symmetry: $x = 0$

P15

Function Properties
- Two x-intercepts
- y-intercept $= 0$
- Line of symmetry: $x = 2$

P16

Function Properties
- One x-intercept
- y-intercept $= 9$
- Line of symmetry: $x = -3$

P17

Function Properties
- Two x-intercepts
- y-intercept $= 0$
- Line of symmetry: $x = 2$

P18

48. Refer to Graphs G7 and G8 from Exercise 47. Without using your calculator, answer the following questions.

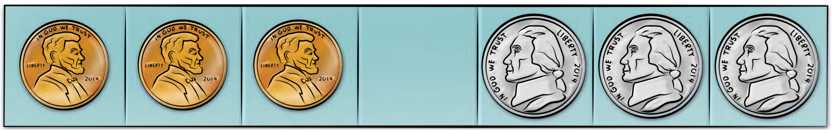
a. Suppose parabola G7 is shifted 1 unit left. Write an equation for this new parabola.

b. Suppose parabola G7 is shifted 4 units right. Write an equation for this new parabola.

c. Can parabola G7 be transformed into parabola G8 by a shift to the right only? Explain.

Extensions

49. A puzzle involves a strip of seven squares, three pennies, and three nickels. The starting setup is shown.



To solve the puzzle, you must switch the positions of the coins so the nickels are on the left and the pennies are on the right. You can move a coin to an empty square by sliding it or by jumping it over one coin. You can move pennies only to the right and nickels only to the left.

You can make variations of this puzzle by changing the numbers of coins and the length of the strip. Each puzzle should have the same number of each type of coin and one empty square.

a. Make drawings that show each move (slide or jump) required to solve puzzles with 1, 2, and 3 coins of each type. How many moves does it take to solve each puzzle?

b. A puzzle with n nickels and n pennies can be solved with $n^2 + 2n$ moves. Use this expression to calculate the number of moves required to solve puzzles with 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 of each type of coin.

c. Do your calculations for 1, 2, and 3 coins of each type from part (b) agree with the numbers you found in part (a)?

d. By calculating first and second differences in the data from part (b), verify that the relationship between the number of moves and the number of each type of coin is quadratic.

Use the following information for Exercises 50–52.

A soccer coach wants to take her 20-player team to the state capital for a tournament. A travel company is organizing the trip. The cost will be \$125 per student. The coach thinks this is too expensive, so she decides to invite other students to go along. For each extra student, the cost of the trip will be reduced by \$1 per student.

50. The travel company's expenses for the trip are \$75 per student. The remaining money is profit. What will the company's profit be if the following numbers of students go on the trip?

a. 20 **b.** 25 **c.** 60 **d.** 80

51. Let n represent the number of students who go on the trip. In parts (a)–(d), write an equation for the relationship described. It may help to make a table like the one shown here.

State Capital Trip

Number of Students	Price per Student	Travel Company's Income	Travel Company's Expenses	Travel Company's Profit
20	\$125	20 × \$125 = \$2,500	20 × \$75 = \$1,500	\$2,500 – \$1,500 = \$1,000
21	\$124	■	■	■

a. the relationship between *the price per student* and n

b. the relationship between *the travel company's income* and n

c. the relationship between *the travel company's expenses* and n

d. the relationship between *the travel company's profit* and n

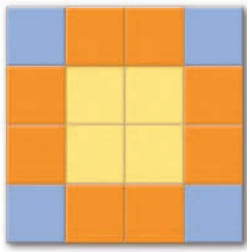
52. Use a calculator to make a table and a graph of the equation for the travel company's profit. Study the pattern of change in the profit as the number of students increases from 25 to 75.

a. What number of students gives the company the maximum profit?

b. What numbers of students guarantee the company will earn a profit?

c. What numbers of students will give the company a profit of at least \$1,200?

53. The Terryton Tile Company makes floor tiles. One tile design uses grids of small, colored squares as in this 4×4 pattern.



a. Suppose you apply the same design rule to a 5×5 pattern. How many small squares will be blue? How many will be yellow? How many will be red?

b. How many small squares of each color will there be if you apply the rule to a 10×10 pattern?

c. How many small squares of each color will there be if you apply the rule to an $n \times n$ pattern?

d. What kinds of relationships between the side length of the pattern and the number of small squares of each color do the expressions in part (c) describe? Explain.

54. This prism is made from centimeter cubes. After the prism was built, its faces were painted. How many centimeter cubes have

a. no painted faces?

b. one painted face?

c. two painted faces?

d. three painted faces?

e. How many centimeter cubes are there in all?