Applications

1. a. At 5 seconds, the flare will have traveled to a maximum height of 400 ft.

   b. The flare will hit the water when the height is 0 ft, which will occur at 10 seconds.

   c. In a graph, the maximum point represents the maximum height of the flare, and the right-hand *x*-intercept represents the point at which the flare hits the water. In a table, the entry for when the height is its greatest represents the maximum height reached by the flare, and the entry for when the height is once again 0 represents the point at which the flare hits the water.

2. a. The rocket will travel to a height of 148 feet. It reaches this maximum height after 2 seconds.

   b. The rocket was launched at a height of 84 feet above ground level.

   c. It will take 4 seconds for the rocket to return to the height from which it was launched.

3. a. The ball is released at about 6.5 ft (the *y*-intercept).

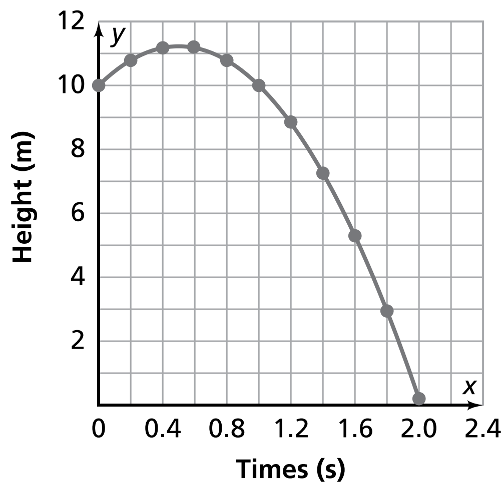
   b. The ball reaches its maximum height, about 17.5 ft, at about 0.8 seconds.

   c. The ball would reach the basket just after 1.5 seconds.

4. a. **Height of a Diver after *t* Seconds**

Time (t)	Height (h)
0	10
0.2	10.784
0.4	11.176
0.6	11.176
0.8	10.784
1.0	10
1.2	8.824
1.4	7.256
1.6	5.296
1.8	2.944
2.0	0.2
2.2	−2.936

   b. **Diving From the Platform**



c. The diver hits the water's surface when the height is 0, which happens between 2 and 2.1 seconds. In the graph, this is the x-intercept. In the table, it is the entry for when height is 0.

d. The diver will be 5 m above the water's surface between 1.6 and 1.7 seconds.

e. The diver is falling at the greatest rate just before hitting the water's surface. In the table, this is when the difference of successive height values is the greatest. In the graph, this is where the curve has the steepest downward slope.

5. a. The maximum height is about 15.06 ft, which occurs after about 0.56 seconds. **Note:** Students can find this by making a table or a graph of the equations.

b. Her feet hit the water when the height is 0, which occurs at about 1.53 seconds.

c. The board is 10 ft above the water's surface.

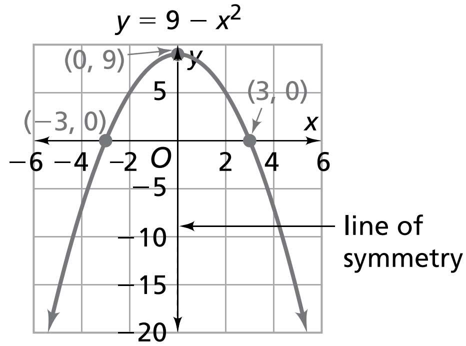
6. a. The maximum height is 44 ft, which is reached at 1.5 seconds. You could find this in a table of time versus height by locating the maximum height. You could find this in a graph by determining the height at the maximum point of the parabola.

b. The ball hits the ground just after 3.1 seconds. You could find this in a table of time versus height by locating the value for time when height is 0. You could find this in a graph by determining the time at the point at which the parabola crosses the x-axis.

c. The ball begins rising rapidly and then slows its ascent until it reaches the maximum height of 44 ft. It then starts to fall, slowly at first and gaining speed on the way down until it hits the ground.

d. The ball is 8 ft above ground when thrown.

7.



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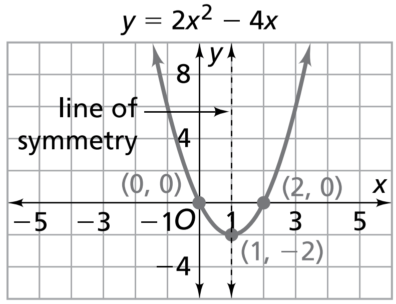

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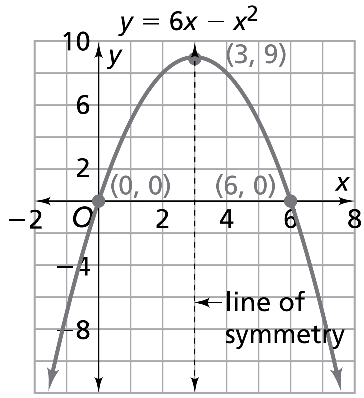

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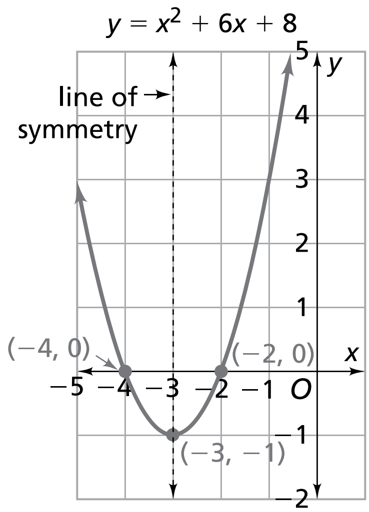


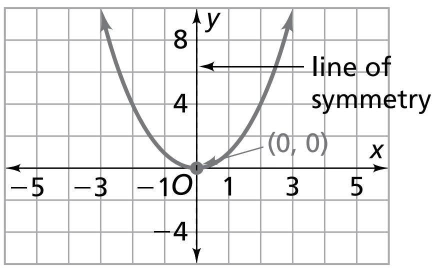
11. a. If the sign of the coefficient of the x^2 term is negative, the graph will have a maximum point. If it is positive, the graph will have a minimum point.

b. The x-intercepts are the values that make each factor in the factored form of the equation equal to 0. The y-intercept is the constant term in the expanded form of the equation.

c. If there are two x-intercepts, the distances from each x-intercept to the line of symmetry are the same. If there is only one intercept, it is on the line of symmetry. There is not any apparent relationship between the y-intercept and the line of symmetry.

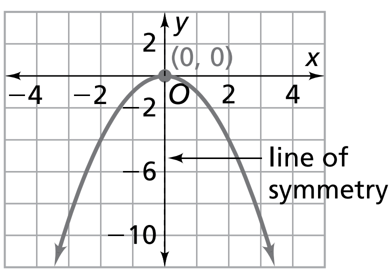
12. We can predict that this is a parabola with x- and y-intercepts and minimum at (0, 0).

$y = x^2$



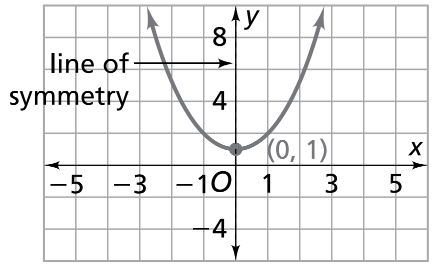
13. We can predict that this is a parabola with x- and y-intercepts and maximum at (0, 0).

$y = -x^2$



14. We can predict that this is a parabola with a minimum, and the y-intercept at (0, 1).

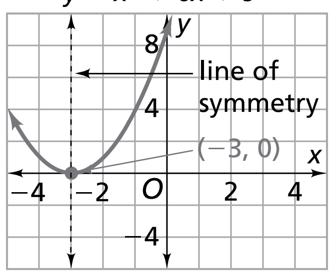
$y = x^2 + 1$



Note: This graph does not have real roots; that is, it does not cross the x-axis. If $y = 0$, then $x^2 = -1$, so x is a complex number.

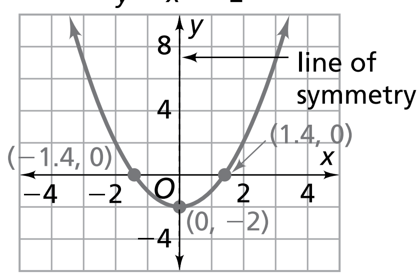
15. If we factor $y = x^2 + 6x + 9$, we have $y = (x + 3)^2$. From this, we can predict this is a parabola with minimum and x-intercept at $(-3, 0)$. We can predict the y-intercept from $y = x^2 + 6x + 9$; it is (0, 9).

$y = x^2 + 6x + 9$



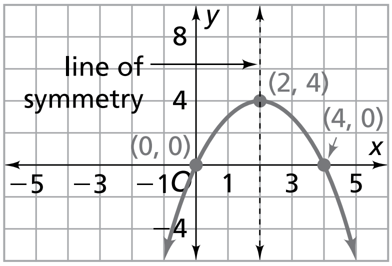
16. We can predict that this is a parabola with a minimum and y-intercept at $(0, -2)$.

$y = x^2 - 2$



17. We can predict that this is a parabola with x-intercepts at 0 and 4, and a vertex at (2, 4). From the expanded form $y = 4x - x^2$ we can predict there will be a maximum at (2, 4).

$y = x(4 - x)$



18. **a.** the 8 corners, or 8 cubes

b. The cubes along the 12 edges that are not corner cubes, or $12 \times 10 = 120$ cubes.

c. The large cube has 6 faces, and each face contains $10 \times 10 = 100$ cubes with one face painted, a total of $6 \times 100 = 600$ cubes.

d. Removing the external cubes leaves $10 \times 10 \times 10 = 1{,}000$ unpainted cubes.

19. **a.** The unpainted cubes form a 10-by-10-by-10 cube on the inside of the large cube, which means the dimensions of the large cube must be 12 by 12 by 12, with 1,728 total cubes. Of these, 1,000 have no painted faces, 600 are painted on 1 face, 120 are painted on 2 faces, and 8 are painted on 3 faces.

b. Each of the 6 faces on the cube contains $\frac{864}{6} = 144$ cubes with one face painted. There are 144 cubes arranged in a 12-by-12 square, which means the large cube must have the dimensions of 14 by 14 by 14, with 2,744 total cubes. Of these, 1,728 have no painted faces, 864 are painted on 1 face, 144 are painted on 2 faces, 8 are painted on 3 faces.

c. Each of the 12 edges contains $\frac{132}{12} = 11$ cubes painted on two faces, which means the large cube must have the dimensions of 13 by 13 by 13, with 2,197 total cubes. Of these, 1,331 have paint on no faces, 726 are painted on 1 face, 132 are painted on 2 faces, 8 are painted on 3 faces.

d. Any cube would have 8 cubes painted on three faces, located at the 8 corners; we cannot tell the size of the large cube based on this information.

20. **a.** In the values for x, first differences are constant. In the values for x^2, second differences are constant. In the values for x^3, the third differences are constant.

x	x
1	1
2	2
3	3
4	4
5	5

x	x^2
1	1
2	4
3	9
4	16
5	25

x	x^3
1	1
2	8
3	27
4	64
5	125

b. In the table of value x, the pattern of change is similar to the pattern of the number of cubes with 3 or 2 faces painted because their first differences are constant. In the table of value x^2, the pattern of change is similar to the pattern of the number of cubes with 1 face painted because their second differences are constant. In the table of value x^3, the pattern of change is similar to the pattern of the number of cubes with 0 faces painted because their third differences are constant.

21. $y_1 = 2(x - 1)$ is similar to the relationship of the number of cubes painted on two faces because they are both linear. $y_2 = (x - 1)^3$ is similar to the relationship of the number of cubes painted on 0 faces or total cubes because they are both cubic. $y_3 = 4(x - 1)^2$ is similar to the relationship for the number of cubes painted on one face because they are both quadratic. **Note:** Students can observe the similarity from the form of equations or the pattern of changes in tables.

22–25. The line of symmetry can be found by finding the point on the x-axis that is halfway between the x-intercepts. If this point is a, then the line of symmetry is $x = a$. The x-intercepts can be read directly from the factored form or estimated from the graph of a quadratic equation.

22. B; Line of symmetry: $x = -\frac{9}{2}$

23. D; Line of symmetry: $x = -\frac{3}{2}$

24. C; Line of symmetry: $x = -1$

25. A; Line of symmetry: $x = 0$

26. **a.** Possible answer: They both have the same x-intercepts and they both have the same axis of symmetry.

b. Possible answer: One opens up and the other opens down. One has a maximum and the other has a minimum.

c. (5, 25); substituting $x = 5$ into $y = x(10 - x)$ produces $y = 25$.

d. (5, −25); substituting $x = 5$ into $y = x(x - 10)$ produces $y = -25$.

27. D

Connections

28. **a.** Table 1: Each y-value is twice the previous y-value. The missing entry is (5, 800).
Table 2: Each y-value is 3 greater than the previous y-value. The missing entry is (2, 18).
Table 3: Each increase in the y-value is 2 greater than the previous increase. The missing entry is (7, 56).
Table 4: Each increase in the y-value is 2 less than the previous increase. The missing entry is (3, 16).

b. Table 1; $y_6 = 25(2)^x$
Table 2; $y_5 = 3(x + 4)$
Table 3; $y_2 = x(x + 1)$
Table 4; $y_3 = 25 - x^2$

c. Tables 3 and 4; in Tables 3 and 4, the second differences are constant.

d. Table 4; (0, 25).

e. None of the tables shows a minimum y-value. Because the table has only a few entries, there is a least y-value among those listed. This least y-value is not necessarily the least y-value of the entire function. Only the pattern for the function represented in Table 3 has a minimum y-value, but it is not listed in the table.

29. **a.** The equations are equivalent. Possible explanation: When you graph the equations, the graphs are identical, so the equations must be the same. You can also use the Distributive Property to show that the equations are equivalent.

b. Possible answers: This equation is not equivalent to the other two because its graph is different. Or, substituting the same value for p into all three equations proves that they are not equivalent. For example, substituting 20 for p gives the following values for I:

$I = (100 - p)p = (100 - 20)20 = (80)20 = 1{,}600$.

$I = 100p - p^2 = 100(20) - 20^2 = 2{,}000 - 400 = 1{,}600$.

$I = 100 - p^2 = 100 - 20^2 = 100 - 400 = -300$.

c. $M = (100 - p)p - 350$, or $M = 100p - p^2 - 350$.

d. A price of \$50 gives the maximum profit, which is \$2,150. **Note:** This can be seen in a graph or a table of the equation as shown below. (See Figure 1.)

Profit From Art Fair

Price (\$)	Profit (\$)
10	550
20	1,250
30	1,750
40	2,050
50	2,150
60	2,050
70	1,750
80	1,250
90	550

e. For prices under about \$3.65 and over about \$96.35, the potter will lose money, so the potter will make a profit on prices between these amounts. **Note:** These points are the *x*-intercepts; students can approximate them by making a table or a graph.

30. **a.** $12 \times 12 = 144$ eggs in each layer.

b. $144 \times 12 = 1{,}728$ eggs in the container

31. **a.** $A = x^2$; $P = 4x$

b. $A = (2x)^2 = 4x^2$, so the area would increase by a factor of 4. $P = 4(2x) = 8x$, so the perimeter would increase by a factor of 2. **Note:** Students may solve this by testing several examples.

c. $A = (3x)^2 = 9x^2$, so the area would increase by a factor of 9. Since $P = 4(3x) = 12x$, the perimeter would increase by a factor of 3.

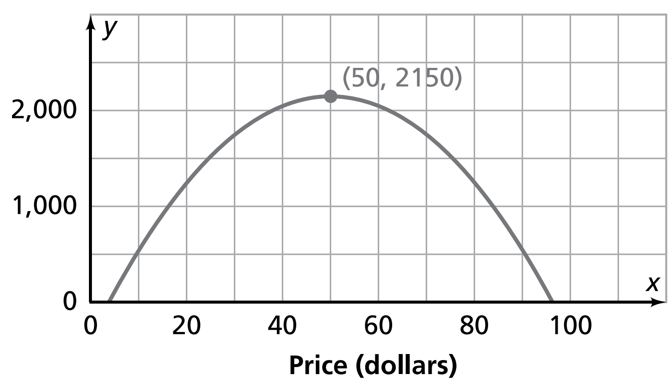
d. Since $A = 36$ m^2, $x = 6$ m, so $P = 4(6) = 24$ m.

e. **Side Length, Perimeter, and Area of a Square**

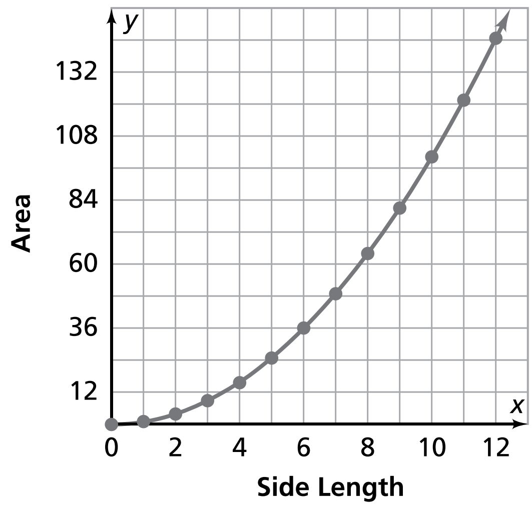
x	$4x$	x^2
0	0	0
1	4	1
2	8	4
3	12	9
4	16	16
5	20	25
6	24	36
7	28	49
8	32	64
9	36	81
10	40	100
11	44	121
12	48	144

Figure 1

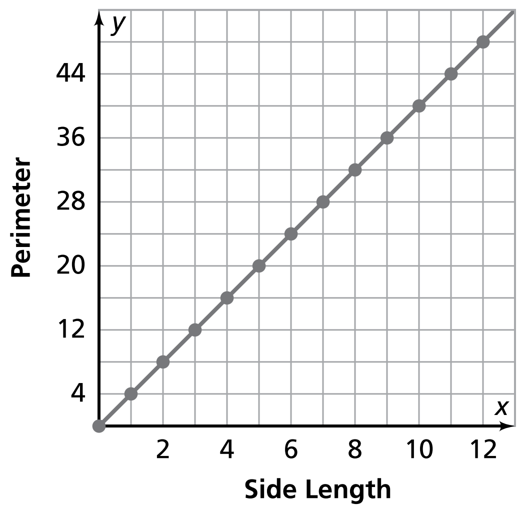
Profit From Art Fair



f. **(x, x^2)**



(x, $4x$)



g. The relationship is quadratic between the side length (x) and the area (x^2).The relationship is linear between the side length (x) and the perimeter ($4x$).

32. **a.** $V = x^3$

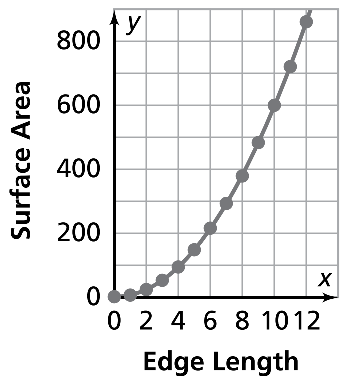
b. $V = (2x)^3 = 8x^3$; the volume would increase by a factor of 8.

c. $V = (3x)^3 = 27x^3$; S.A. $= 6(3x)^2 = 6(9x^2) = 9(6x^2)$; the volume would increase by a factor of 27 and the surface area would increase by a factor of 9.

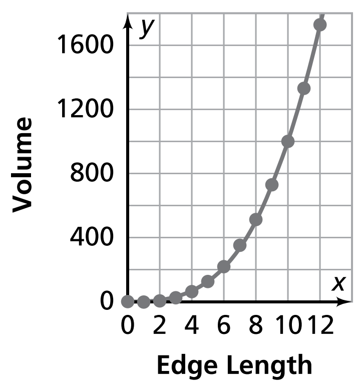
Length, Surface Area, and Volume of a Cube

Edge Length	Surface Area	Volume
0	0	0
1	6	1
2	24	8
3	54	27
4	96	64
5	150	125
6	216	216
7	294	343
8	384	512
9	486	729
10	600	1,000
11	726	1,331
12	864	1,728

d. **Edge Length vs. Surface Area**



e. **Edge Length vs. Volume**



f. The patterns of change in the table and graph for surface area suggest the relationship between length and area is quadratic. The patterns of change in the table and graph for volume suggest the relationship between length and volume is cubic.

33. a. The areas are π square cm and 4π square cm.

b. The relationship is quadratic. The area increases by increasing amounts. Students might examine the differences of areas, or they might graph the radii and area to see if they get a quadratic, or they might use symbols to justify that $y = 3.14x^2$ is a quadratic relationship. (See Figure 2.)

c. The length of the smaller rectangle is the same as the circumference of the smaller circle or 2π. So the surface area of the smaller cylinder is $\pi + \pi + (2\pi)(2)$, or 6π square units. The surface area of the larger cylinder is $4\pi + 4\pi + 4\pi\ (2)$ or 16π square units.

d. Yes; students might examine second differences and see that they are a constant 2π. Or they might identify the equation $y = 2\pi x(x + 2)$ as the equation of a parabola with x-intercepts at 0 and -2. (See Figure 3.)

34. B

35. a. (See Figure 4.)

Figure 2

Relationship of a Radius to Area of a Circle

Radius	1	2	3	4	x
Area	π	4π	9π	16π	πx^2

Figure 3

Surface Areas of Cylinders With Different Radius and Height

Radius	1	2	3	4	x
Height	2	2	2	2	2
Surface Area	6π	16π	$[9 + 9 + (6)(2)]\pi = 30\pi$	$[16 + 16 + (8)(2)]\pi = 48\pi$	$\pi x^2 + \pi x^2 + (2\pi x)(2) = 2\pi x^2 + 4\pi x = 2\pi x(x + 2)$

Figure 4

Surface Areas of Cylinders With Equal Radius and Height

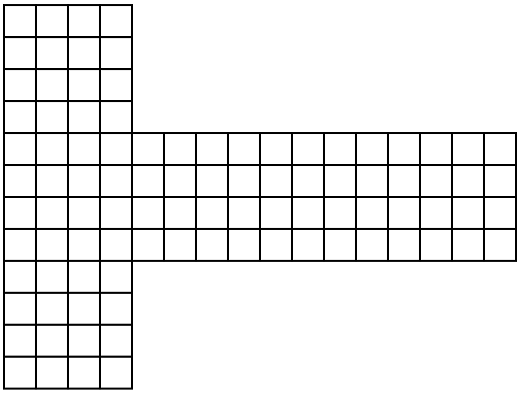
Radius	1	2	3	4	x
Height	1	2	3	4	x
Surface Area	$\pi + \pi + (2\pi)(1) = 4\pi$	$4\pi + 4\pi + (4\pi)(2) = 16\pi$	$9\pi + 9\pi + (6\pi)(3) = 36\pi$	$16\pi + 16\pi + (8\pi)(4) = 64\pi$	$\pi x^2 + \pi x^2 + (2\pi x)(x) = 4\pi x^2$

b. Yes; students might examine the second differences and see that they are a constant 8π, or they might identify the relationship's equation of $y = 4\pi x^2$ as the equation of a parabola.

36. a. Each edge is 3 units.

b. The surface area is 54 square units. The volume is 27 cubic units.

c. Student drawings should show the flat pattern of a cube with edge 4 units, surface area 6(16) or 96 square units. There are many such nets. One possibility is shown.



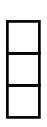
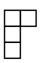
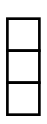
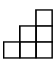
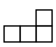
d. $V = x^3$.This is not quadratic (it is a cubic relationship). Students might make a table and examine how the volume grows, or they might graph $y = x^3$ and examine the shape, or they might refer to the symbols.

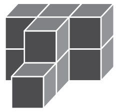
37. No; the surface area of Silvio's box is 1,536 square inches, since $16^2 \times 6 = 1{,}536$. Ten square feet of wrapping paper equals 1,440 square inches since a square foot is 144 square inches and 10(144) = 1,440. There will not be enough paper.

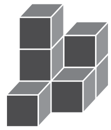
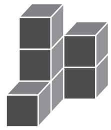
38. H

39. C

40.

Building	1	2	3	4
Base				
Front				
Right				

41. 

42.  

Building 1 **Building 2**

43. If $x = 5$, then $x(x - 5) = 0$. If $x = -5$, then $x(x - 5) = 50$.

44. If $x = 1$, then $3x^2 - x = 2$. If $x = \frac{1}{3}$, then $3x^2 - x = 0$.

45. If $x = 2$, then $x^2 + 5x + 4 = 18$. If $x = -4$, then $x^2 + 5x + 4 = 0$.

46. If $x = -2$, then $(x - 7)(x + 2) = 0$. If $x = 2$, then $(x - 7)(x + 2) = -20$.

47. y_1 matches Graph 9 and Property Card 15
y_2 matches Graph 11 and Property Card 16 or 18
y_3 matches Graph 10 and Property Card 13
y_4 matches Graph 12 and Property Card 17
y_5 matches Graph 8 and Property Card 16 or 18
y_6 matches Graph 7 and Property Card 14

Note: It may be worth discussing which properties are sufficient to make a parabola unique. For example, Graphs 8 and 11 have the same y-intercept, same x-intercepts, and the same line of symmetry, but they are different graphs. Had the line of symmetry been replaced with the vertex (max. or min. point), it would be enough to make the parabola unique.

48. a. The equation was $y = x(x + 4)$. The new graph would have x-intercepts at $x = -5$ and $x = -1$, so the equation would be $y = (x + 1)(x + 5)$.

b. The equation was $y = x(x + 4)$. The new graph would have x-intercepts at $x = 0$ and $x = 4$, so the equation would be $y = x(x - 4)$.

c. If you translate parabola G7 to the right by 4 units, its *x*-intercepts would coincide with those of parabola G8. However, their orientations are different: parabola G7 opens upward, while parabola G8 opens downward. So, you would also need to reflect parabola G7 over the *x*-axis in order to transform it into parabola G8.

Extensions

49. a. It takes 3 moves to solve the puzzle with 1 pair of coins. Starting with

1		5

the moves could be as follows:

	1	5

5	1	

5		1

It takes 8 moves to solve the puzzle with 2 pairs of coins. Starting with

1	1		5	5

the moves could be as follows:

1		1	5	5

1	5	1		5

1	5	1	5	

1	5		5	1

	5	1	5	1

5		1	5	1

5	5	1		1

5	5		1	1

It takes 15 moves to solve the puzzle with 3 pairs of coins. Starting with

1	1	1		5	5	5

the moves could be as follows:

1	1	1	5		5	5

1	1		5	1	5	5

1		1	5	1	5	5

1	5	1		1	5	5

1	5	1	5	1		5

1	5	1	5	1	5	

1	5	1	5		5	1

1	5		5	1	5	1

	5	1	5	1	5	1

5		1	5	1	5	1

5	5	1		1	5	1

5	5	1	5	1		1

5	5	1	5		1	1

5	5		5	1	1	1

5	5	5		1	1	1

b. (See Figure 6.)

c. The number of moves calculated from the expression agree with the numbers found above. (See Figure 7.)

d. Second differences are a constant 2 (See Figure 6.), so the relationship is quadratic.

50. a. If only the 20 soccer team members go, the cost of the trip is \$125 per student. The travel agent's profit is the difference of income and cost, or $P = 125n - 75n$, where n is the number of students:
$P = 125(20) - 75(20) = 2{,}500 - 1{,}500 =$ \$1,000.

b. If 25 students go, the cost is \$120 per student and the agent's profit is
$P = 120n - 75n = 120(25) - 75(25 =$
$3{,}000 - 1{,}875 =$ \$1,125.

c. If 60 students go, the cost is \$85 per student and the agent's profit is
$P = 85n - 75n = 85(60) - 75(60) =$
$5{,}100 - 4{,}500 =$ \$600.

d. If 80 students go, the cost is \$65 per student and the agent's profit is
$P = 65n - 75n = 65(80) - 75(80) =$
$5{,}200 - 6{,}000 = -$\$800.

For this many students, the travel agent would lose money.

Figure 6

Number of Each Type of Coin	Number of Moves
1	3
2	8
3	15
4	24
5	35
6	48
7	63
8	80
9	99
10	120

First Differences	Second Differences
5	2
7	2
9	2
11	2
13	2
15	2
17	2
19	2
21	

Figure 7

# of Type of Coin	1	2	3	4	5	6	7	8	9	10
# of Moves	3	8	15	24	35	48	63	80	99	120

51. (See Figure 8.)

a. price per student = $125 - (n - 20)$, or $125 - n + 20$, or $145 - n$

b. income = price $\times\ n = [125 - (n - 20)]n$, or $125n - n(n - 20)$, or $125n - n^2 + 20n$, or $145n - n^2$

c. expenses = $75n$

d. profit = income − expenses = $[125 - (n - 20)]n - 75n$, or $125n - n(n - 20) - 75n$, or $-n^2 + 125n + 20n - 75n$, or $70n - n^2$.

52. **a.** The agent's profit is greatest for 35 students.

b. If fewer than 70 students go on the trip, the agent will make a profit.

c. From 30 students to 40 students give the travel agent a profit of at least $1,200.

53. **a.** blue: 4
yellow: $3 \times 3 = 9$
red: $4 \times 3 = 12$

b. blue: 4
yellow: $8 \times 8 = 64$
red: $4 \times 8 = 32$

c. blue: 4
yellow: $(n - 2)^2$
red: $4(n - 2)$

d. The relationship described by $S = (n - 2)^2$ is quadratic because it is formed by the product of two linear factors. The relationship $S = 4(n - 2)$ is linear.

54. **a.** 0 cubes

b. 6 cubes

c. 16 cubes

d. 8 cubes

e. $8 + 16 + 6 = 30$ cubes, or $3 \times 2 \times 5 = 30$ cubes

Figure 8

Pricing and Profit Scenarios for a Travel Agent

Number of Students	Price per Student	Travel Agent's Income	Travel Agent's Expenses	Travel Agent's Profits
20	125	$20 \times 125 = 2{,}500$	$20 \times 75 = 1{,}500$	$2{,}500 - 1{,}500 = 1{,}000$
21	124	$21 \times 124 = 2{,}604$	$21 \times 75 = 1{,}575$	$2{,}604 - 1{,}575 = 1{,}029$
22	123	$22 \times 123 = 2{,}706$	$22 \times 75 = 1{,}650$	$2{,}706 - 1{,}650 = 1{,}056$
23	122	$23 \times 122 = 2{,}806$	$23 \times 75 = 1{,}725$	$2{,}806 - 1{,}725 = 1{,}081$