Applications

1. These dot patterns represent the first four *square numbers,* 1, 4, 9, and 16.

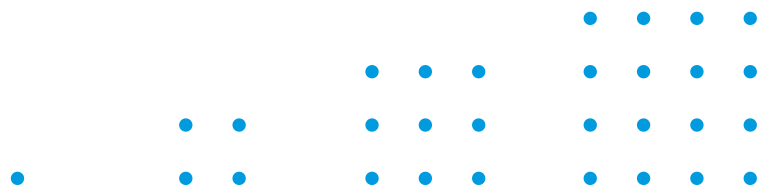


Figure 1 Figure 2 Figure 3 Figure 4

a. What are the next two square numbers?

b. Write an equation for the nth square number s.

c. Make a table and a graph of (n, s) values for the first ten square numbers. Describe the pattern of change from one square number to the next.

2. The numbers of dots in the figures below are the first four *rectangular numbers*.

Figure 1 Figure 2 Figure 3 Figure 4

a. What are the first four rectangular numbers?

b. Find the next two rectangular numbers.

c. Describe the pattern of change from one rectangular number to the next.

d. Predict the seventh and eighth rectangular numbers.

e. Write an equation for the nth rectangular number r.

3. In Problem 3.1, you looked at triangular numbers.

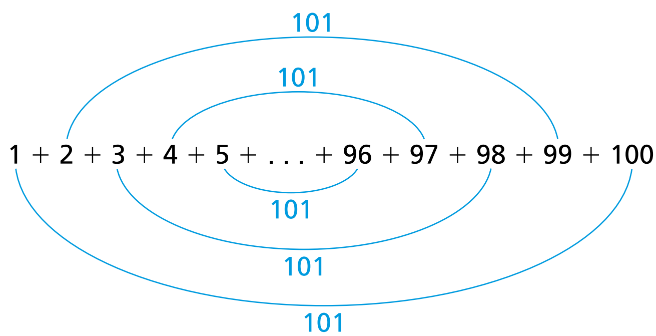
   a. What is the 18th triangular number?

   b. Is 210 a triangular number? Explain.

Did You Know?

Carl Friedrich Gauss (1777–1855) was a German mathematician and astronomer. When Gauss was about eight years old, his teacher asked his class to find the sum of the first 100 counting numbers. Gauss had the answer almost immediately!

Gauss realized that he could pair up the numbers as shown. Each pair has a sum of 101.



There are 100 numbers, so there are 50 pairs. This means the sum is $50 \times 101 = 5{,}050$ $\left[\text{or } \frac{100}{2}(101) \text{ or } \frac{100}{2}(\text{first number plus last number})\right]$.

4. a. In Problem 3.1, you found an equation for the *n*th triangular number. Sam claims he can use this equation to find the sum of the first ten counting numbers. Explain why Sam is correct.

   b. What is the sum of the first ten counting numbers?

   c. What is the sum of the first 15 counting numbers?

   d. What is the sum of the first *n* counting numbers?

For Exercises 5–8, tell whether the number is a triangular number, a square number, a rectangular number, or none of these. Explain.

5. 110 **6.** 66 **7.** 121 **8.** 60

9. In a middle school math league, each team has six student members and two coaches.

a. At the start of a match, the coaches and student members of one team exchange handshakes with the coaches and student members of the other team. How many handshakes occur?

b. At the end of the match, the members and coaches of the winning team exchange high fives. How many high fives occur?

c. The members of one team exchange handshakes with their coaches. How many handshakes occur?

10. In a 100-meter race, five runners are from the United States and three runners are from Canada.

a. How many handshakes occur if the runners from one country exchange handshakes with the runners from the other country?

b. How many high fives occur if the runners from the United States exchange high fives?

11. A company rents five offices in a building. There is a cable connecting each pair of offices.

a. How many cables are there in all?

b. Suppose the company rents two more offices. How many cables will they need in all?

c. Compare this situation with Case 3 in Problem 3.2.

For Exercises 12–15, describe a situation that can be represented by the equation. Tell what the variables p and n represent in that situation.

12. $p = n(n - 1)$

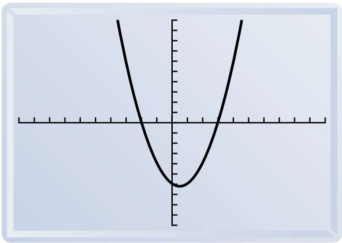
13. $p = 2n$

14. $p = n(n - 2)$

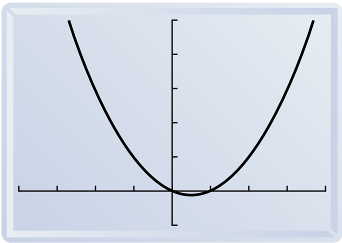
15. $p = n(16 - n)$

16. The graphs below represent situations you have looked at in this Unit.

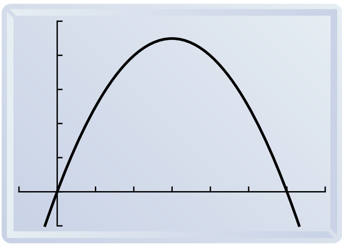
Graph I



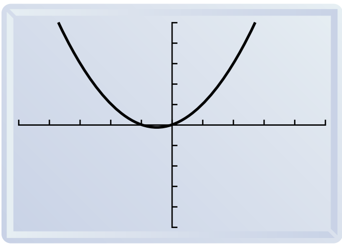
Graph II



Graph III



Graph IV



a. Which graph might represent the number of high fives exchanged among a team with n players? Explain.

b. Which graph might represent the areas of rectangles with a fixed perimeter?

c. Which graph might represent the areas of a rectangle formed by increasing one dimension of a square by 2 centimeters and decreasing the other dimension by 3 centimeters?

d. Which graph might represent a triangular-number pattern?

For Exercises 17–19, the tables represent quadratic functions. Copy and complete each table.

17.

x	y
0	0
1	1
2	3
3	6
4	■
5	■
6	■

18.

x	y
0	0
1	3
2	8
3	15
4	■
5	■
6	■

19.

x	y
0	0
1	4
2	6
3	6
4	■
5	■
6	■

For Exercises 20–24, tell whether the table represents a quadratic function. If it does, tell whether the function has a maximum or minimum value.

20.

x	–3	–2	–1	0	1	2	3	4	5
y	–4	1	4	5	4	1	–4	–11	–18

21.

x	0	1	2	3	4	5	6	7	8
y	2	3	6	11	18	27	38	51	66

22.

x	0	1	2	3	4	5	6	7	8
y	0	–4	–6	–6	–4	0	6	14	24

23.

x	–4	–3	–2	–1	0	1	2	3	4
y	5	4	3	2	1	2	3	4	5

24.

x	–4	–3	–2	–1	0	1	2	3	4
y	18	10	4	0	–2	–2	0	4	10

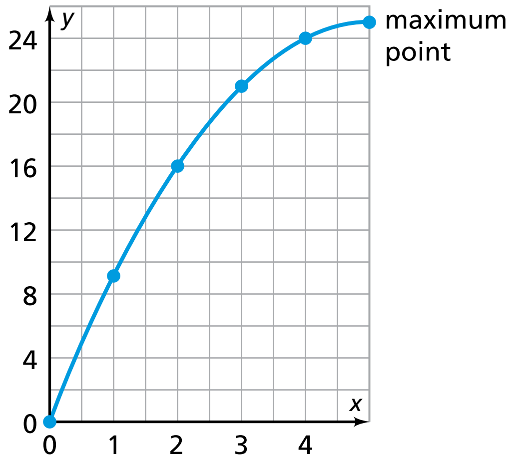
25. **a.** For each equation, investigate the pattern of change in the y-values as the x-values increase or decrease at a constant rate. Describe the patterns you find.

$y = 2x^2$ $\quad$ $y = 3x^2$ $\quad$ $y = \frac{1}{2}x^2$ $\quad$ $y = -2x^2$

b. Use what you discovered in part (a) to predict the pattern of change for each of these equations.

$y = 5x^2$ $\quad$ $y = -4x^2$ $\quad$ $y = \frac{1}{4}x^2$ $\quad$ $y = ax^2$

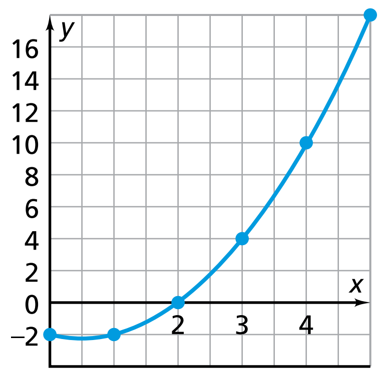
26. Use the graph below.



a. Make a table of (x, y) values for the six points shown on the graph.

b. The graph shows a quadratic function. Extend the graph to show x-values from 5 to 10. Explain how you know your graph is correct.

27. The graph shows a quadratic function. Extend the graph to show x-values from -4 to 0.

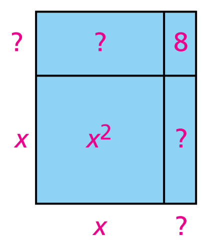


28. The table below shows a quadratic function. Extend the table to show x-values from 0 to -5. Explain how you know your table is correct.

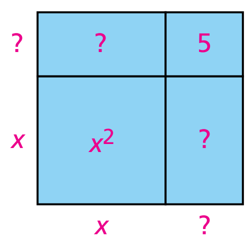
x	y
0	8
1	3
2	0
3	−1
4	0
5	3

Connections

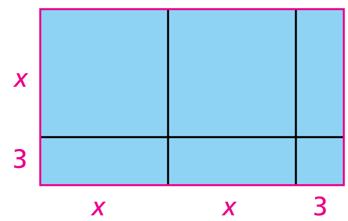
29. **a.** Make sketches that show two ways of completing the rectangle model below using whole numbers. For each sketch, express the area of the large rectangle in both expanded form and factored form.



b. Is there more than one way to complete the rectangle model below using whole numbers? Explain.



30. Write two equivalent expressions for the area of the rectangle outlined in red below.



31. Consider these quadratic expressions.

$2x^2 + 7x + 6$ $\qquad\qquad$ $x^2 + 6x + 8$

a. For each expression, sketch a rectangle whose area represents the expression. Which expression is easier to present in a rectangle model? Why?

b. Write each expression in factored form.

For Exercises 32–37, write the expression in expanded form.

32. $x(5 - x)$ **33.** $(x + 1)(x + 3)$ **34.** $(x - 1)(x + 3)$

35. $3x(x + 5)$ **36.** $(2x + 1)(x + 3)$ **37.** $(2x - 1)(x + 3)$

For Exercises 38–43, write the expression in factored form.

38. $x^2 - 9x + 8$ **39.** $4x^2 - 6x$ **40.** $3x^2 + 14x + 8$

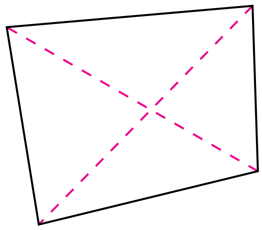
41. $4x^2 + 6x$ **42.** $4x^2 - x - 3$ **43.** $x^3 - 2x^2 - 3x$

44. Min was having trouble factoring the expression in Exercise 40. Ricardo suggested that she use a rectangle model.

a. Explain how a rectangle model can help Min factor the expression. Make a sketch to illustrate your explanation.

b. How you can factor an expression without drawing a rectangle?

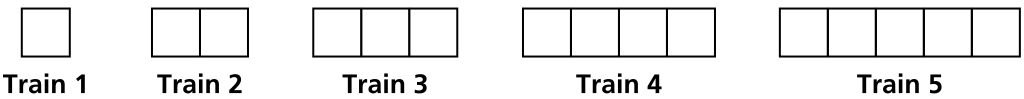
45. A diagonal of a polygon is a line segment connecting any two nonadjacent vertices. A quadrilateral has two diagonals like the one below.



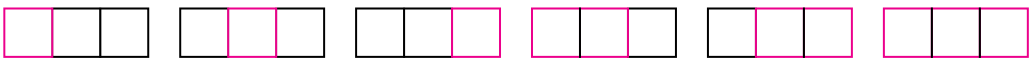
a. How many diagonals does a pentagon have? How many does a hexagon have? A heptagon? An octagon?

b. How many diagonals does an n-sided polygon have?

46. These "trains" are formed by joining identical squares.



a. How many rectangles are in each of the first five trains? For example, the diagram below shows the six rectangles in Train 3. (Remember, a square is a rectangle.)



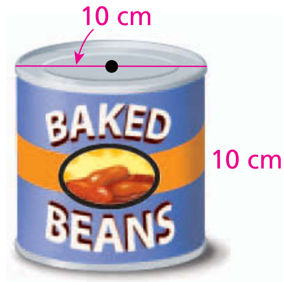
b. Make a table showing the number of rectangles in each of the first ten trains.

c. How can you use the pattern of change in your table to find the number of rectangles in Train 15?

d. Write an equation for the number of rectangles in Train n.

e. Use your equation to find the number of rectangles in Train 15.

47. **a.** What is the area of the base of the can?



b. How many centimeter cubes or parts of cubes can fit in a single layer on the bottom of the can?

c. How many layers of this size would fill the can?

d. Use your answers to parts (a)–(c) to find the volume of the can.

e. The label on the lateral surface of the can is a rectangle with a height of 10 centimeters. What is the other dimension of the label?

f. What is the area of the label?

g. Use your answers to parts (a) and (f) to find the surface area of the can.

48. A company is trying to choose a box shape for a new product. It has narrowed the choices to the triangular prism and the cylinder shown below.

a. Sketch a net for each box.

b. Find the surface area of each box.

c. Which box will require more cardboard to construct?

For Exercise 49–52, tell whether the pattern in each table represents a function that is linear, quadratic, exponential, or none of these.

49.

x	y
0	2
3	4
5	5
6	6
7	7
8	8
10	10

50.

x	y
−3	12
−2	7
−1	4
0	3
1	4
2	7
3	12

51.

x	y
0	1
2	9
5	243
6	729
7	2,187
8	6,561
10	59,049

52.

x	y
1	−2
2	0
3	3
4	8
5	15
6	24
7	14

53. **Multiple Choice** Which equation represents a quadratic relationship?

A. $y = (x - 1)(6 - 2)$

B. $y = 2x(3 - 2)$

C. $y = 2^x$

D. $y = x(x + 2)$

54. **Multiple Choice** Which equation has a graph with a minimum point at (1, 4)?

F. $y = -x^2 + 5$

G. $y = -x^2 + 5x$

H. $y = x^2 - 2x + 5$

J. $y = -x^2 + 7x - 10$

55. Write each expression in expanded form.

a. $-3x(2x - 1)$

b. $1.5x(6 - 2x)$

Extensions

56. You can use Gauss's method to find the sum of the whole numbers from 1 to 10 by writing the sum twice as shown and adding vertically.

$$\begin{array}{r}
1 + \ \ 2 + \ \ 3 + \ \ 4 + \ \ 5 + \ \ 6 + \ \ 7 + \ \ 8 + \ \ 9 + 10 \\
\underline{10 + \ \ 9 + \ \ 8 + \ \ 7 + \ \ 6 + \ \ 5 + \ \ 4 + \ \ 3 + \ \ 2 + \ \ 1} \\
11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11
\end{array}$$

Each vertical sum of 11 occurs 10 times, or $10(11) = 110$. This result is twice the sum of the numbers from 1 to 10, so you divide by 2 to get $\frac{10(11)}{2} = \frac{110}{5} = 55$.

a. How can you use this idea to find $1 + 2 + 3 + \ldots + 99 + 100$?

b. How could you use this idea to find $1 + 2 + 3 + \ldots + n$ for any whole number n?

c. How is this method related to Gauss's method?

57. The patterns of dots below represent the first three *star numbers*.

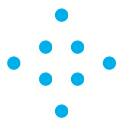


Figure 1

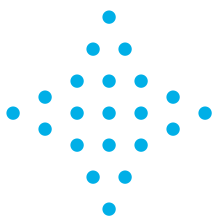


Figure 2

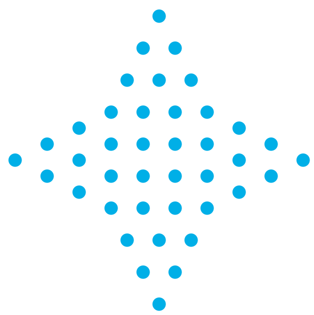


Figure 3

a. What are the first three star numbers?

b. Find the next three star numbers.

c. Write an equation you could use to calculate the nth star number.

58. In parts (a) and (b), explain your answers by drawing pictures or writing a convincing argument.

a. Ten former classmates attend their class reunion. They all shake hands with each other. How many handshakes occur?

b. A little later, two more classmates arrive. Suppose these two people shake hands with each other and the ten other classmates. How many new handshakes occur?

59. The pattern of dots below represents the first three *hexagonal numbers*.

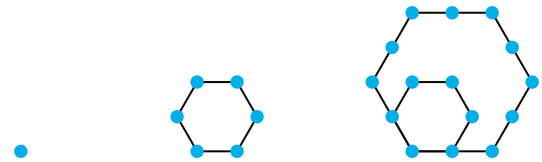


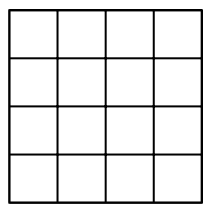
Figure 1 **Figure 2** **Figure 3**

a. What are the first three hexagonal numbers?

b. Find the next two hexagonal numbers.

c. Write an equation you can use to calculate the nth hexagonal number.

60. There are 30 squares of various sizes in this 4-by-4 grid.



a. Sixteen of the squares are the identical small squares that make up the grid. Find the other 14 squares. Draw pictures or give a description.

b. How many squares are in an n-by-n grid? (**Hint:** Start with some simple cases and search for a pattern.)

61. Complete parts (a) and (b) for each equation.

$y_1 = x + 1$ $\qquad$ $y_2 = (x + 1)(x + 2)$

$y_3 = (x + 1)(x + 2)(x + 3)$ $\qquad$ $y_4 = (x + 1)(x + 2)(x + 3)(x + 4)$

a. Describe the shape of the graph of the equation. Include any special features.

b. Describe the pattern of change between the variables.