Applications

1. A square has sides of length x centimeters. One dimension increases by 4 centimeters and the other decreases by 4 centimeters, forming a new rectangle.

a. Make a table showing the side length and area of the square and the area of the new rectangle. Include whole-number x-values from 4 to 16.

b. On the same axes, graph the data (x, *area*) for both the square and the rectangle.

c. Suppose you want to compare the area of a square with the area of the corresponding new rectangle. Is it easier to use the table or the graph?

d. Write equations for the area of the original square and the area of the new rectangle in terms of x.

e. Use your calculator to graph both equations. Show values of x from -10 to 10. Copy the graphs onto your paper. Describe the relationship between the two graphs.

2. A square has sides of length x centimeters. One dimension increases by 5 centimeters, forming a new rectangle.

a. Make a sketch to show the new rectangle.

b. Write two expressions, one in factored form and one in expanded form, for the area of the new rectangle.

c. Choose one of your expressions from part (b). Use it to write an equation for the area A of the new rectangle in terms of x. Then, graph the equation.

For Exercises 3 and 4, draw a divided rectangle whose area is represented by each expression. Label the lengths and area of each section. Then, write an equivalent expression in expanded form.

3. $x(x + 7)$

4. $x(x - 3)$

For Exercises 5–7, draw a divided rectangle whose area is represented by each expression. Label the lengths and area of each section. Then, write an equivalent expression in factored form.

5. $x^2 + 6x$

6. $x^2 - 8x$

7. $x^2 - x$

For Exercises 8–11, write the expression in factored form.

8. $x^2 + 10x$

9. $x^2 - 6x$

10. $x^2 + 11x$

11. $x^2 - 2x$

For Exercises 12–15, write the expression in expanded form.

12. $x(x + 1)$

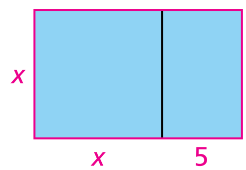
13. $x(x - 10)$

14. $x(x + 6)$

15. $x(x - 15)$

For Exercises 16–20, write two expressions, one in factored form and one in expanded form, for the area of the rectangle outlined in red.

16.



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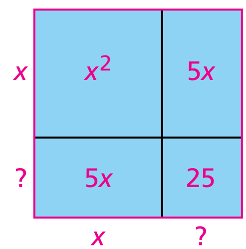

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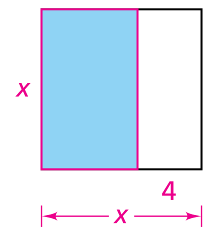

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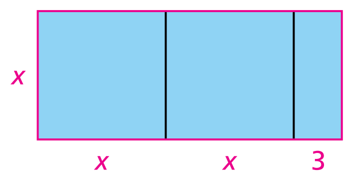

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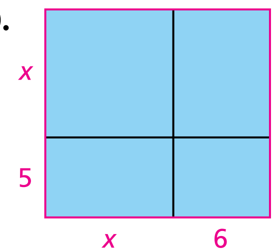


21. A square has sides of length x meters. Both dimensions increase by 5 meters, forming a new square.

a. Make a sketch to show the new square.

b. Write two expressions, one in factored form and one in expanded form, for the area of the new square.

c. Choose one of your expressions from part (b). Use it to write an equation for the area A of the new square in terms of x. Then, graph the equation. Does the equation represent a quadratic function? Explain.

22. A square has sides of length x centimeters. One dimension increases by 4 centimeters and the other increases by 5 centimeters, forming a new rectangle.

a. Make a sketch to show the new rectangle.

b. Write two expressions, one in factored form and one in expanded form, for the area of the new rectangle.

c. Choose one of your expressions from part (b). Use it to write an equation for the area A of the new square in terms of x. Then, graph the equation. Does the equation represent a quadratic function? Explain.

For Exercises 23–34, use the Distributive Property to write each expression in expanded form.

23. $(x - 3)(x + 4)$	**24.** $(x + 3)(x + 5)$	**25.** $x(x + 5)$
26. $(x - 2)(x - 6)$	**27.** $(x - 3)(x + 3)$	**28.** $(x - 3)(x + 5)$
29. $(2x + 1)(x + 1)$	**30.** $(x - 1)(7x + 1)$	**31.** $(x - 1)(3x - 3)$
32. $(x + 7)^2$	**33.** $(3x + 4)^2$	**34.** $(3x - 4)^2$

35. a. Draw and label a rectangle whose area is represented by each expression.

$x^2 + 3x + 4x + 12$ $\qquad$ $x^2 + 7x + 10$

b. For each expression in part (a), write an equivalent expression in factored form.

36. Write each expression in factored form.

a. $x^2 + 13x + 12$	**b.** $x^2 - 13x + 12$	**c.** $x^2 + 8x + 12$
d. $x^2 - 8x + 12$	**e.** $x^2 + 7x + 12$	**f.** $x^2 - 7x + 12$
g. $x^2 + 11x - 12$	**h.** $x^2 - 11x - 12$	**i.** $x^2 + 4x - 12$
j. $x^2 - 4x - 12$	**k.** $x^2 + x - 12$	**l.** $x^2 - x - 12$

37. Write each expression in expanded form. Look for a pattern. Make a generalization about the expanded form of expressions of the form $(x + a)(x + a)$.

a. $(x + 1)(x + 1)$ **b.** $(x + 5)(x + 5)$ **c.** $(x - 5)(x - 5)$

38. Write each expression in expanded form. Look for a pattern. Make a generalization about the expanded form of expressions of the form $(x + a)(x - a)$.

a. $(x + 1)(x - 1)$ **b.** $(x + 5)(x - 5)$ **c.** $(x + 1.5)(x - 1.5)$

39. Use your generalizations from Exercises 37 and 38 to write each of these expressions in factored form.

a. $x^2 + 6x + 9$ **b.** $x^2 - 6x + 9$

c. $x^2 - 9$ **d.** $x^2 - 16$

40. Write each expression in factored form.

a. $2x^2 + 5x + 3$ **b.** $4x^2 - 9$ **c.** $4x^2 + 12x + 9$

41. Write each difference of squares in factored form.

a. $x^2 - 49$ **b.** $4x^2 - 49$ **c.** $25x^2 - 1.44$

For Exercises 42–50, determine whether the equation represents a quadratic function *without* making a table or a graph. Explain.

42. $y = 5x + x^2$ **43.** $y = 2x + 8$ **44.** $y = (9 - x)x$

45. $y = 4x(3 + x)$ **46.** $y = 3^x$ **47.** $y = x^2 + 10x$

48. $y = x(x + 4)$ **49.** $y = 2(x + 4)$ **50.** $y = 7x + 10 + x^2$

51. Rewrite each equation in expanded form. Then, give the x- and y-intercepts, the coordinates of the maximum or minimum point, and the line of symmetry for the graph of each equation.

a. $y = (x - 3)(x + 3)$ **b.** $y = x(x + 5)$ **c.** $y = (x + 3)(x + 5)$

d. $y = (x - 3)(x + 5)$ **e.** $y = (x + 3)(x - 5)$ **f.** $y = x(x - 3)$

For Exercises 52 and 53, complete parts (a)–(e).

a. Find an equivalent factored form of the equation.

b. Identify the x- and y-intercepts for the graph of the equation.

c. Find the coordinates of the maximum or minimum point.

d. Find the line of symmetry.

e. Tell which form of the equation can be used to predict the features in parts (b)–(d) without making a graph.

52. $y = x^2 + 5x + 6$

53. $y = x^2 - 25$

54. Darnell makes a rectangle from a square by doubling one dimension and adding 3 centimeters. He leaves the other dimension unchanged.

a. Write an equation for the area A of the new rectangle in terms of the side length x of the original square.

b. Graph your area equation.

c. What are the x-intercepts of the graph? How can you find the x-intercepts from the graph? How can you find them from the equation?

Connections

55. The winner of the Jammin' Jelly jingle contest will receive \$500. Antonia and her friends are writing a jingle. They plan to divide the prize money equally if they win.

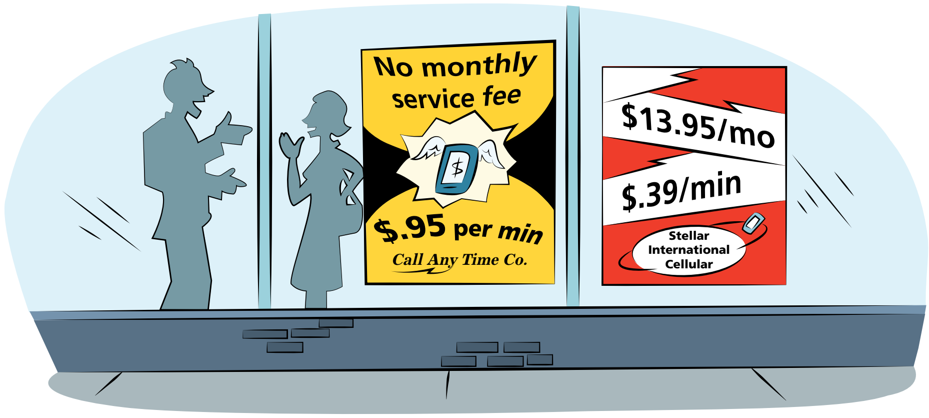
a. Suppose n friends write the winning jingle. How much prize money will each person receive?

b. Describe the relationship between the number of friends and the prize money each friend receives.

c. Write a question about this relationship that is easier to answer by making a graph. Write a question that is easier to answer by making a table. Write a question that is easier to answer by writing an equation.

d. Is this relationship a quadratic function, a linear function, an exponential function, or an inverse variation? Explain.

56. The Stellar International Cellular long-distance company and the Call Any Time company have different charge plans.



a. Represent each charge plan with an equation, a table, and a graph.

b. For each plan, tell whether the relationship between calling time and monthly cost is a quadratic function, a linear function, an exponential function, or an inverse variation. How do your equation, table, and graph support your answer?

c. For what number of minutes are the costs for the two plans equal?

57. A square has sides of length x centimeters.

a. The square is enlarged by a scale factor of 2. What is the area of the enlarged square?

b. How does the area of the original square compare with the area of the enlarged square?

c. Is the new square similar to the original square? Explain.

58. A rectangle has dimensions of x centimeters and $(x + 1)$ centimeters.

a. The rectangle is enlarged by a scale factor of 2. What is the area of the enlarged rectangle?

b. How does the area of the original rectangle compare with the area of the enlarged rectangle?

c. Is the new rectangle similar to the original rectangle? Explain.

59. Suppose the circumference of a cross section of a tree is x feet.

a. What is the diameter in terms of x?

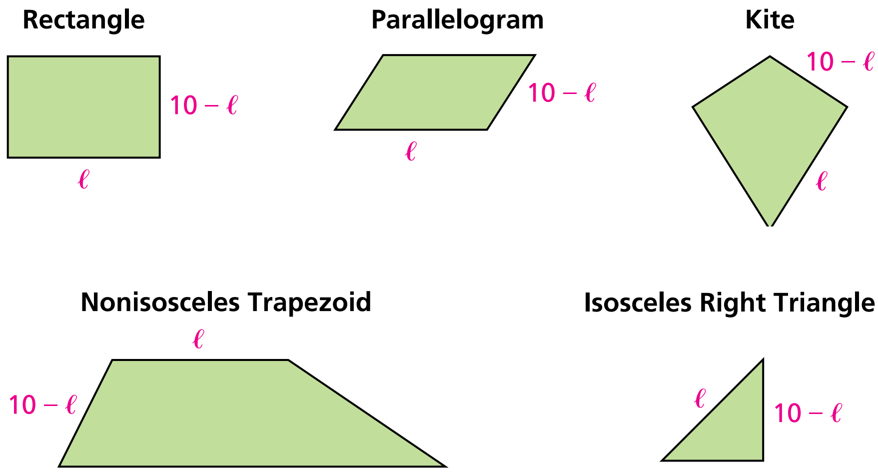
b. What is the radius in terms of x?

c. What is the area of the cross section in terms of x?

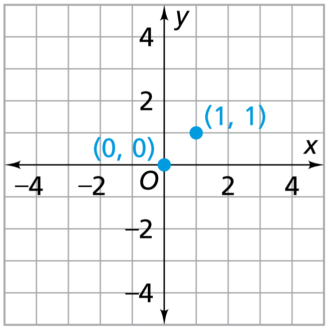
d. Is the relationship between the circumference and the area of the cross section linear, quadratic, exponential, or none of these?

e. Suppose the circumference of the cross section is 10 feet. What are the diameter, radius, and area of the cross section?

60. For each polygon, write formulas for the perimeter P and area A in terms of ℓ if it is possible. If it is not possible to write a formula, explain why.



61. **a.** Write the equation of the line that passes through the two points shown.



b. Is there a different line that can be drawn through these points? Explain.

For Exercises 62–65, evaluate the expression for the given values of *x*.

62. $x(x - 5)$ for $x = 2$ and $x = 3$

63. $3x^2 - x$ for $x = 1$ and $x = \frac{1}{3}$

64. $x^2 + 5x + 4$ for $x = 2$ and $x = -4$

65. $(x - 7)(x + 2)$ for $x = -2$ and $x = 2$

Extensions

66. **Multiple Choice** Which expression is equivalent to $(2n + 3)(4n + 2)$?

A. $8n + 5$

B. $6n^2 + 7n + 4n + 5$

C. $8n^2 + 16n + 6$

D. $8n^2 + 6$

For Exercises 67 and 68, write each expression in factored form. You may want to draw a rectangle model.

67. $2x^2 + 3x + 1$

68. $4x^2 + 10x + 6$

69. Sketch graphs of the equations $y = x^2 + 2x$ and $y = x^2 + 2$.

a. How are the graphs similar?

b. How are the graphs different?

c. Find the y-intercept for each graph.

d. Find the x-intercepts for each graph if they exist. If there are no x-intercepts, explain why.

e. Do all quadratic functions have y-intercepts? Explain.