Applications

1. a.

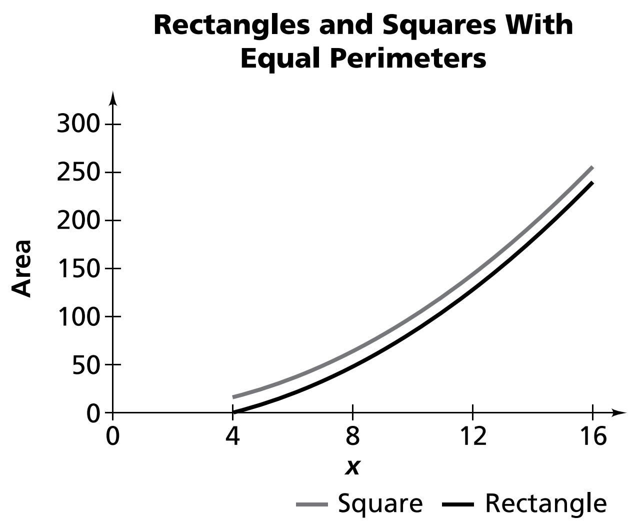
Square		Rectangle		
Side	Area	Length	Width	Area
4	16	8	0	0
5	25	9	1	9
6	36	10	2	20
7	49	11	3	33
8	64	12	4	48
9	81	13	5	65
10	100	14	6	84
11	121	15	7	105
12	144	16	8	128
13	169	17	9	153
14	196	18	10	180
15	225	19	11	209
16	256	20	12	240

b. (See Figure 1.)

c. The graph and the table both show that the area of the rectangle increases as the area of the square increases. The area of the square is always 16 cm^2 greater than the area of the rectangle. This constant difference of the two can be seen on the graph, but the table shows the exact value of the difference.

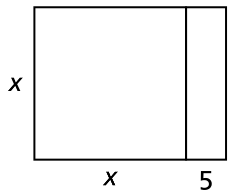
d. Area of the square is $A = x^2$ where x is the side length and the area of the new rectangle is $A = (x + 4)(x - 4)$ or $A = x^2 - 16$.

Figure 1



e. (See Figure 2 and Figure 3.)

2. a.



b. $A = x(x + 5)$ and $A = x^2 + 5x$

c.

$y = x(x + 5)$

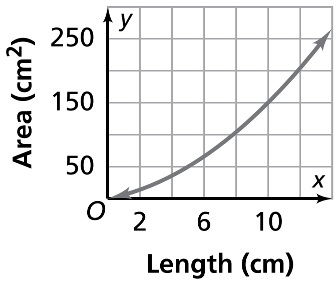


Figure 2

Area of a Square

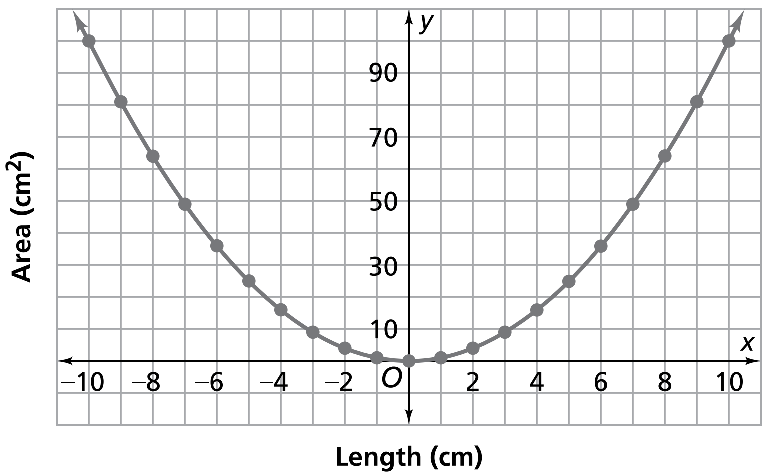
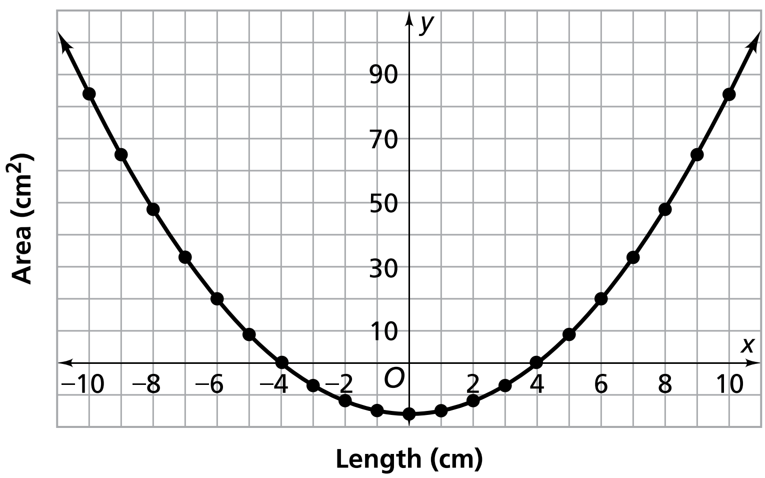
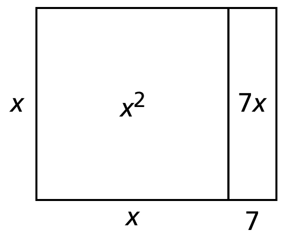


Figure 3

Area of a Rectangle

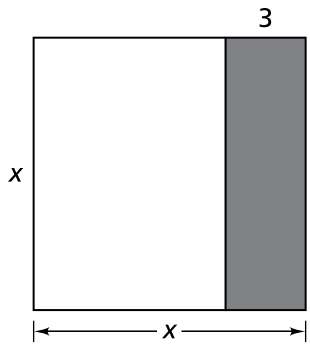


3. $x^2 + 7x$

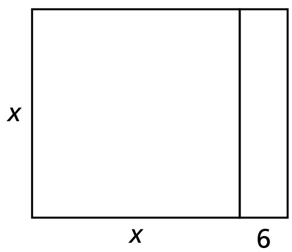


Note: In Exercises 4, 6, and 7, students may reverse the shaded/unshaded portions of the square.

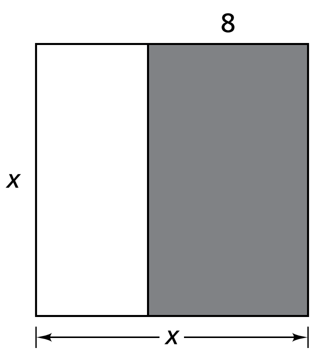
4. $x^2 - 3x$



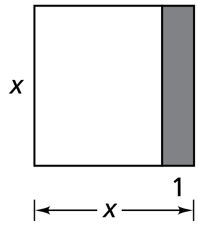
5. $x(x + 6)$



6. $x(x - 8)$



7. $x(x - 1)$



8. $x(x + 10)$

9. $x(x - 6)$

10. $x(x + 11)$

11. $x(x - 2)$

12. $x^2 + x$

13. $x^2 - 10x$

14. $x^2 + 6x$

15. $x^2 - 15x$

16. $x(x + 5)$ and $x^2 + 5x$

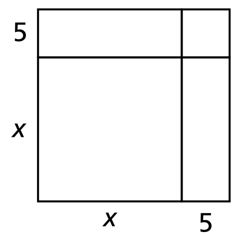
17. $x^2 + 5x + 5x + 25$ and $(x + 5)(x + 5)$

18. $x(x - 4)$ and $x^2 - 4x$

19. $x(2x + 3)$ and $x^2 + x^2 + 3x$

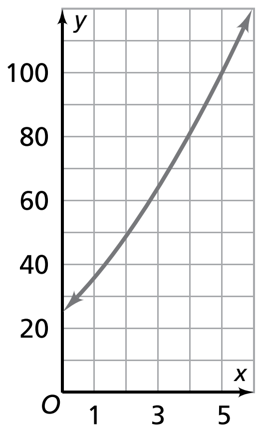
20. $(x + 5)(x + 6)$ and $x^2 + 5x + 6x + 30$

21. **a.**



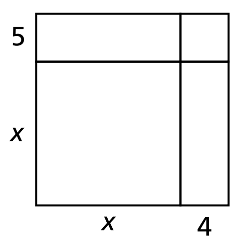
b. $(x + 5)(x + 5)$ or $x^2 + 5x + 5x + 25$, which is equivalent to $x^2 + 10x + 25$

c. $A = x^2 + 10x + 25$; this equation is quadratic because 2 is the highest power of x.



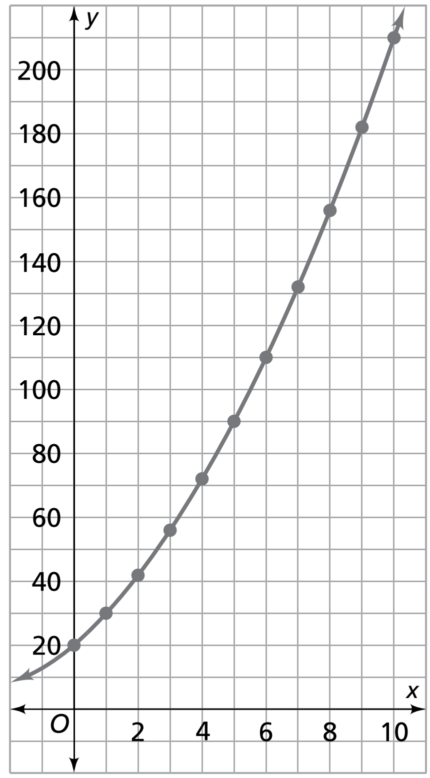
Note: To see the parabola shape we need a window which includes negative values which could not, in practical terms, represent lengths.

22. a.



b. $A = (x + 5)(x + 4)$ and $A = x^2 + 5x + 4x + 20$.

c. This equation is quadratic because 2 is the highest power of x.



Note: To see the parabola shape we need a window that includes negative values which could not, in practical terms, represent lengths.

23. $x^2 - 3x + 4x - 12$ or $x^2 + x - 12$

24. $x^2 + 3x + 5x + 15$ or $x^2 + 8x + 15$

25. $x^2 + 5x$

26. $x^2 - 2x - 6x + 12$ or $x^2 - 8x + 12$

27. $x^2 - 3x + 3x - 9$, or $x^2 - 9$

28. $x^2 - 3x + 5x - 15$, or $x^2 + 2x - 15$

29. $2x^2 + x + 2x + 1$ or $2x^2 + 3x + 1$

30. $7x^2 - 7x + x - 1$ or $7x^2 - 6x - 1$

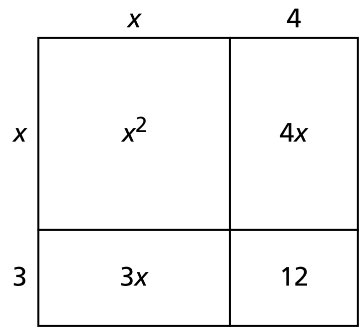
31. $3x^2 - 3x - 3x + 3$ or $3x^2 - 6x + 3$

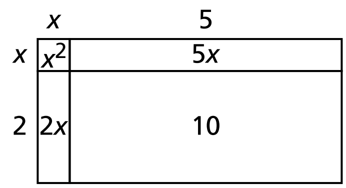
32. $(x + 7)(x + 7) = x^2 + 7x + 7x + 49$ or $x^2 + 14x + 49$

33. $(3x + 4)(3x + 4) = 9x^2 + 12x + 12x + 16$ or $9x^2 + 24x + 16$

34. $(3x - 4)(3x - 4) = 9x^2 - 12x - 12x + 16$ or $9x^2 - 24x + 16$

35. a.





b. $(x + 3)(x + 4)$
$(x + 2)(x + 5)$

36. a. $(x + 12)(x + 1)$;
$$\begin{aligned} x^2 + 13x + 12 &= x^2 + 12x + 1x + 12 \\ &= x(x + 12) + 1(x + 12) \\ &= (x + 12)(x + 1) \end{aligned}$$

b. $(x - 12)(x - 1)$;
$$\begin{aligned} x^2 - 13x + 12 &= x^2 - 12x - 1x + 12 \\ &= x(x - 12) + (-1)(x - 12) \\ &= (x - 12)(x - 1) \end{aligned}$$

c. $(x + 6)(x + 2)$;
$$\begin{aligned} x^2 + 8x + 12 &= x^2 + 6x + 2x + 12 \\ &= x(x + 6) + 2(x + 6) \\ &= (x + 6)(x + 2) \end{aligned}$$

d. $(x - 6)(x - 2)$;
$$\begin{aligned} x^2 - 8x + 12 &= x^2 - 6x - 2x + 12 \\ &= x(x - 6) + (-2)(x - 6) \\ &= (x - 6)(x - 2) \end{aligned}$$

e. $(x + 3)(x + 4)$;
$$\begin{aligned} x^2 + 7x + 12 &= x^2 + 3x + 4x + 12 \\ &= x(x + 3) + 4(x + 3) \\ &= (x + 3)(x + 4) \end{aligned}$$

f. $(x - 3)(x - 4)$;
$$\begin{aligned} x^2 - 7x + 12 &= x^2 - 3x - 4x + 12 \\ &= x(x - 3) + (-4)(x - 3) \\ &= (x - 3)(x - 4) \end{aligned}$$

g. $(x + 12)(x - 1)$;
$$\begin{aligned} x^2 + 11x + 12 &= x^2 + 12x - 1x + 12 \\ &= x(x + 12) + (-1)(x + 12) \\ &= (x + 12)(x - 1) \end{aligned}$$

h. $(x - 12)(x + 1)$;
$$\begin{aligned} x^2 - 11x + 12 &= x^2 - 12x + 1x + 12 \\ &= x(x - 12) + 1(x - 12) \\ &= (x - 12)(x + 1) \end{aligned}$$

i. $(x + 6)(x - 2)$;
$$\begin{aligned} x^2 + 4x + 12 &= x^2 + 6x - 2x + 12 \\ &= x(x + 6) + (-2)(x + 6) \\ &= (x + 6)(x - 2) \end{aligned}$$

j. $(x - 6)(x + 2)$;
$$\begin{aligned} x^2 - 4x + 12 &= x^2 - 6x + 2x + 12 \\ &= x(x - 6) + 2(x - 6) \\ &= (x - 6)(x + 2) \end{aligned}$$

k. $(x + 4)(x - 3)$;
$$\begin{aligned} x^2 + x + 12 &= x^2 + 4x - 3x + 12 \\ &= x(x + 4) + (-3)(x + 4) \\ &= (x + 4)(x - 3) \end{aligned}$$

l. $(x - 4)(x + 3)$;
$$\begin{aligned} x^2 - x + 12 &= x^2 - 4x + 3x + 12 \\ &= x(x - 4) + 3(x - 4) \\ &= (x - 4)(x + 3) \end{aligned}$$

37. a. $(x + 1)(x + 1) = x^2 + 2x + 1$

b. $(x + 5)(x + 5) = x^2 + 10x + 25$

c. $(x - 5)(x - 5) = x^2 - 10x + 25$.
The pattern is squaring a binomial when the coefficient of x is 1, so the pattern has the form $(x + a)^2$. The square of a binomial is the square of x plus $2(a)(x)$ plus the square of a. Symbolically, this is represented by:
$(x + a)^2 = (x + a)(x + a) = x^2 + ax + ax + a^2$ or $x^2 + 2ax + a^2$.
A similar pattern holds when the coefficient of x is not 1.
$(ax + c)^2 = (ax + c)(ax + c) = (ax)^2 + acx + acx + c^2$ or $(ax)^2 + 2acx + c^2$.
Students explored this pattern in Problem 2.3.

38. a. $(x + 1)(x - 1) = x^2 - 1$

b. $(x + 5)(x - 5) = x^2 - 25$

c. $(x + 1.5)(x - 1.5) = x^2 - 2.25$

The pattern is multiplying the sum and difference of two numbers. The result is the difference of the squares of the two numbers. Symbolically, this is represented by:
$(x + a)(x - a) = x^2 + ax - ax - a^2$
or $x^2 - a^2$.
A similar pattern holds when the coefficient of x is not 1:
$(ax + c)(ax - c) = (ax)^2 - c^2$.
Students explored this pattern in Problem 2.3.

39. **a.** $x^2 + 6x + 9 = (x + 3)^2$

b. $x^2 - 6x + 9 = (x - 3)^2$

c. $x^2 - 9 = (x + 3)(x - 3)$

d. $x^2 - 16 = (x + 4)(x - 4)$

40. **a.** $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

b. $4x^2 - 9 = (2x + 3)(2x - 3)$

c. $4x^2 + 12x + 9 = (2x + 3)(2x + 3)$

41. **a.** $x^2 - 49 = (x - 7)(x + 7)$

b. $4x^2 - 49 = (2x - 7)(2x + 7)$

c. $25x^2 - 1.44 = (5x - 1.2)(5x + 1.2)$

42. Quadratic; since it has an x^2 term and this is the highest power of x.

43. Not quadratic; it is linear.

44. Quadratic; because it is the product of two linear factors, neither of which is constant.

45. Quadratic; it is the product of two linear factors, neither of which is constant.

46. Not quadratic; it is exponential.

47. Quadratic; since it has an x^2 term and this is the highest power of x.

48. Quadratic; it is the product of two linear factors, neither of which is constant.

49. Not quadratic; it is linear.

50. Quadratic; since it has an x^2 term and this is the highest power of x.

51. **a.** $y = x^2 - 9$; x-intercepts: 3 and -3; y-intercept: -9; Minimum: $(0, -9)$; Line of symmetry: $x = 0$

b. $y = x^2 + 5x$; x-intercepts: 0 and -5; y-intercept: 0; Minimum: $\left(\frac{-5}{2}, \frac{-25}{4}\right)$; Line of symmetry: $x = \frac{-5}{2}$

c. $y = x^2 + 8x + 15$; x-intercepts: -3 and -5; y-intercept: 15; Minimum: $(-4, -1)$; Line of symmetry: $x = -4$

d. $y = x^2 + 2x - 15$; x-intercepts: 3 and -5; y-intercept: -15; Minimum: $(-1, -16)$; Line of symmetry: $x = -1$

e. $y = x^2 - 2x - 15$; x-intercepts: -3 and 5; y-intercept: -15; Minimum: $(1, -16)$; Line of symmetry: $x = 1$

f. $y = x^2 - 3x$; x-intercepts: 0 and 3; y-intercept: 0; Minimum: $(1.5, -2.25)$; Line of symmetry: $x = 1.5$

52. **a.** $y = (x + 3)(x + 2)$

b. y-intercept: 6; x-intercepts: -3 and -2

c. Minimum: $(-2.5, -0.25)$

d. $x = -2.5$

e. The factored form can be useful in predicting the x-intercepts and the axis of symmetry. The expanded form can be useful in predicting the y-intercept. Students may have different preferences in equation forms; however, they should be able to justify their choices.

53. **a.** $y = (x + 5)(x - 5)$

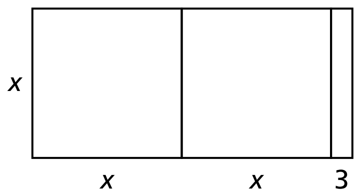
b. y-intercept: -25; x-intercepts: -5 and 5

c. Minimum: $(0, -25)$

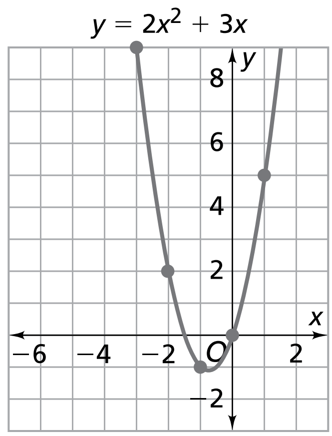
d. $x = 0$

e. The factored form can be useful in predicting the x-intercepts and the axis of symmetry. The expanded form can be useful in predicting the y-intercept. Students may have different preferences in equation forms; however, they should be able to justify their choices.

54. a. Students may choose to draw a rectangle to help them answer this problem. They can represent the area as $A = x(2x + 3)$.



b.



c. The x-intercepts are (0, 0) and $\left(-\frac{3}{2}, 0\right)$.

To find the x-intercept on a graph you find the point(s) where the parabola hits the x-axis. To determine the x-intercepts from the equation, find the values for x that make the factors $2x + 3$ and x equal to zero.

Connections

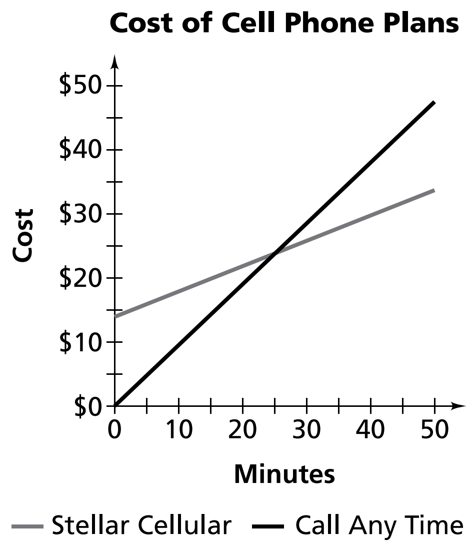
55. a. $P = \frac{500}{n}$

b. This is an inverse relationship: as the number of friends increases, the amount of money each person receives decreases, $n > 0$.

c. A graph would help you answer questions about how the amount of money each person receives changes with the number of people sharing the prize. A table would help answer questions about how much money each person would receive given a specific number of friends. An equation would help answer specific questions about any value of n.

d. This relationship is inverse, which can be seen from the graph or the equation. Students investigated inverse relationships in *Thinking With Mathematical Models.*

56. a. C is the cost for t minutes. Stellar Cellular: $C = 13.95 + 0.39t$, Call Any Time: $C = 0.95t$



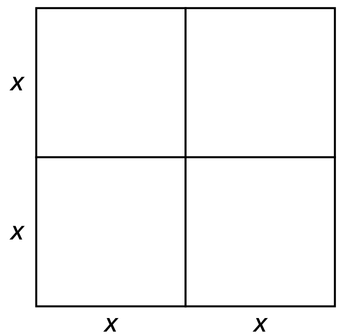
(See Figure 4.)

b. Neither of these plans is quadratic. Both are linear. This can be seen in the equations since t is not multiplied by another factor of t in either equation. Both equations are in the linear form $y = mx + b$. In the table, you can see that both have a constant rate of change, which means they are linear. For the Stellar Cellular plan, the cost increases \$1.95 for every 5 minutes. In the Call Any Time plan, the increase is \$4.75 every 5 minutes. Both graphs look like straight lines, so they are not quadratic.

c. The plans are equal when the number of minutes is about 25 reading from the table. Solving the equation $13.95 + 0.39t = 0.95t$ for t gives an exact answer of about 24.91 minutes.

57. **a.** $A = 2x(2x)$ or $4x^2$

b. The area of the new square is 4 times the area of the original square. Students may choose to make a drawing to help them see this relation between the areas.



c. Yes; the angles are still 90°, and the ratios of pairs of corresponding sides are 2 : 1.

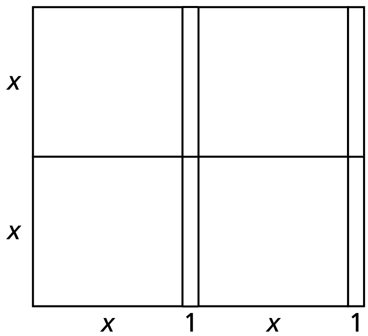
Figure 4

Calls per Minute in Cell Phone Plans

Time in Minutes	Cost in Dollars	
	Stellar Cellular	Call Any Time
0	\$13.95	■
5	\$15.90	\$4.70
10	\$17.85	\$9.50
15	\$19.80	\$14.25
20	\$21.75	\$19.00
25	\$23.70	\$23.75
30	\$25.65	\$28.50
35	\$27.60	\$33.25
40	\$29.55	\$38.00
45	\$31.50	\$42.75
50	\$33.45	\$47.50

58. **a.** $A = 2(x + 1)(2x)$ or $4x^2 + 4x$

b. The area of the new rectangle is 4 times the area of the original rectangle. It can be seen on the drawing below.



c. Yes; the angles are still 90°, and the ratios of pairs of corresponding sides are 2 : 1.

59. **a.** Recall $C = \pi d$, where d is the diameter. So, $x = \pi d$. Or, we can say that $d = \frac{x}{\pi}$.

b. The radius is one half of the diameter, so radius $= \frac{1}{2} \times \frac{x}{\pi}$ or $r = \frac{x}{2\pi}$.

c. $A = \pi r^2$, where r is the radius; $A = \pi\left(\frac{x}{2\pi}\right)^2$.

d. This is a quadratic relation since the x value is squared.

e. $C = 10$ ft, $d = \frac{10}{\pi} \approx 3.18$ feet, $r = \frac{10}{2\pi} = \frac{5}{\pi} \approx 1.59$ feet and $A = \pi\left(\frac{10}{2\pi}\right)^2 \approx$ 7.96 feet.

60. **Rectangle:** $A = \ell(10 - \ell) = 10\ell - \ell^2$ and $P = \ell + \ell(10 - \ell) + (10 - \ell) = 20$

Parallelogram: Area cannot be determined since you are not given the height. $P = 20$.

Symmetric Kite: $P = 20$; area cannot be determined. We can make two triangles by drawing diagonals, but we don't know the bases or heights, so comparing area is not possible.

Nonisosceles Trapezoid: Area and perimeter cannot be determined. Area cannot be determined because you are not given the length of one of the bases or the height. The perimeter cannot be determined because you are not given the length of the other two sides.

Isosceles Right Triangle: Since the triangle is isosceles right, the base is $10 - \ell$ and the height is $10 - \ell$. So, $A = \frac{1}{2}(10 - \ell)(10 - \ell) = 50 - 10\ell + \frac{1}{2}\ell^2$ and $P = \ell + (10 - \ell) + (10 - \ell) = 20 - \ell$.

61. **a.** $y = x$

b. No; given two points, there is only one line that you can draw through them.

62. If $x = 2$, then $x(x - 5) = -6$. If $x = 3$, then $x(x - 5) = -6$.

63. If $x = 1$, then $3x^2 - x = 2$. If $x = \frac{1}{3}$, then $3x^2 - x = 0$.

64. If $x = 2$, then $x^2 + 5x + 4 = 18$. If $x = -4$, then $x^2 + 5x + 4 = 0$.

65. If $x = -2$, then $(x - 7)(x + 2) = 0$. If $x = 2$, then $(x - 7)(x + 2) = -20$.

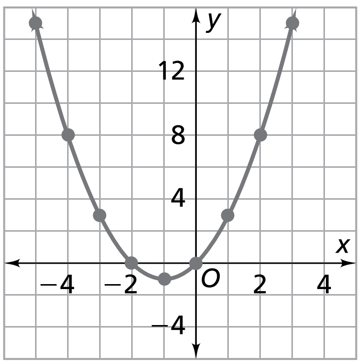
Extensions

66. C

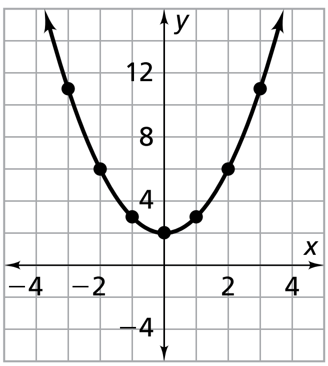
67. $(2x + 1)(x + 1)$

68. $(2x + 3)(2x + 2)$

69. $y = x^2 + 2x$



$y = x^2 + 2$



a. The graphs have the same size and shape. They are both parabolas, and they both open upward.

b. The graphs have different locations on the coordinate plane. They also have different *x*- and *y*-intercepts and lines of symmetry.

c. The *y*-intercept for $y = x^2 + 2x$ is (0, 0). For $y = x^2 + 2$, it is (0, 2).

d. The graph of $y = x^2 + 2x$ has *x*-intercepts of 0 and -2. It is not possible to find the *x*-intercepts for the equation $y = x^2 + 2$ because there is no value of x that you could square and add 2 and get zero.

e. Yes; a parabola will always cross the *y*-axis. If you extend the end of the parabola out to the right and out to the left, eventually it is going to cross the *y*-axis.