Applications

1. Find the maximum area for a rectangle with a perimeter of 120 meters. Make your answer convincing by including these things:

- Sketches of rectangles with a perimeter of 120 meters (Include rectangles that do not have the maximum area and the rectangle you think does have the maximum area.)
- A table of lengths and areas for rectangles with a perimeter of 120 meters (Use increments of 5 meters for the lengths.)
- A graph of the relationship between length and area

Explain how each piece of evidence supports your answer.

2. What is the maximum area for a rectangle with a perimeter of 130 meters? As in Exercise 1, support your answer with sketches, a table, and a graph.

3. The graph shows the length and area of rectangles with a fixed perimeter. Use the graph for parts (a)–(e).

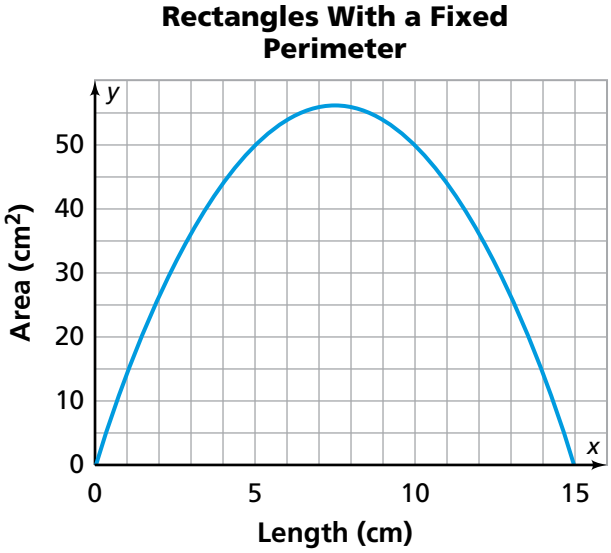
a. Describe the shape of the graph and any special features.

b. What is the maximum area for a rectangle with this fixed perimeter? What are the dimensions of this rectangle?

c. Is there a rectangle with the least possible area? Explain.

d. What is the area of a rectangle with a length of 3 centimeters?

e. Describe two ways to find the fixed perimeter for the rectangles represented by the graph.

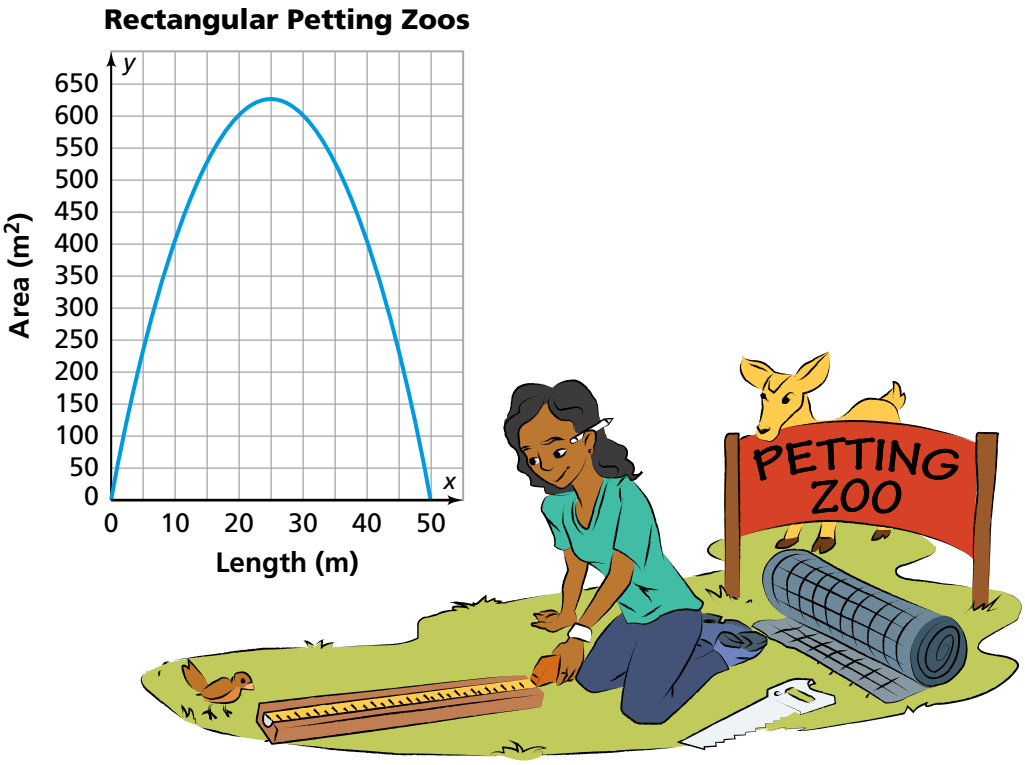


4. Use the graph from Exercise 3. Make a table of values for the length and area.

a. How is the shape of the graph reflected in the table?

b. How can you use the table to find the maximum area and the dimensions of the rectangle with this area? Explain.

5. Hillsdale Farms wants to add a small, rectangular petting zoo for the public. They have a fixed amount of fencing to use for the zoo. This graph shows the lengths and areas of the rectangles they can make.



a. Describe the shape of the graph and any special features you observe.

b. What is the greatest area possible for a rectangle with this perimeter? What are the dimensions of this rectangle?

c. What is the area of the rectangle with a length of 10 meters? What is the area of the rectangle with a length of 40 meters? How are these rectangles related?

d. What are the dimensions of the rectangle with an area of 600 square meters?

e. What is the fixed amount of fencing available for the petting zoo? Explain.

6. The lifeguards at a beach want to place a rectangular boundary around the swimming area that can be used for water basketball. They have a fixed amount of rope to make the boundary. They use the table at the right to look at possible arrangements.

Rectangular Swimming Area

Length (m)	Area (m^2)
1	15
2	28
3	39
4	48
5	55
6	60
7	63
8	64
9	63
10	60
11	55
12	48
13	39
14	28
15	15

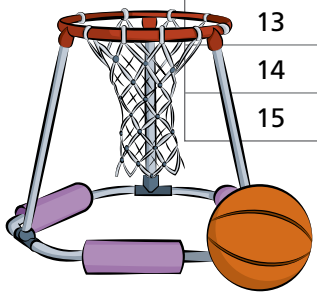
a. What patterns do you observe in the table?

b. What is the fixed perimeter for the possible swimming areas?

c. Sketch a graph of the data (*length, area*). Describe the shape of the graph.

d. Suppose the lifeguards make a rectangle with an area of 11.5 square meters. What are the dimensions of the rectangle?

e. The lifeguards want to enclose the greatest area possible. What should be the dimensions of the swimming area?



7. The equation for the areas of rectangles with a certain fixed perimeter is $A = \ell(20 - \ell)$, where ℓ is the length in meters.

a. Describe the graph of this equation.

b. What is the maximum area for a rectangle with this perimeter? What dimensions correspond to this area? Explain.

c. A rectangle with this perimeter has a length of 15 meters. What is its area?

d. Describe two ways you can find the perimeter. What is the perimeter?

8. A rectangle has a perimeter of 50 meters and a side length of ℓ.

a. Express the other dimension of the rectangle in terms of ℓ.

b. Write an equation for the area A in terms of ℓ.

c. Sketch a graph of your equation and describe its shape.

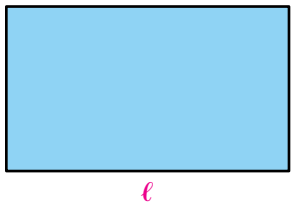
d. Use your equation to find the area of the rectangle with a length of 10 meters.

e. How could you find the area in part (d) by using your graph?

f. How could you find the area in part (d) by using a table?

g. What is the maximum area possible for a rectangle with a perimeter of 50 meters? What are the dimensions of this rectangle?

9. A rectangle has a perimeter of 30 meters and a side length of ℓ.



a. Express the other dimension of the rectangle in terms of ℓ.

b. Write an equation for the area A in terms of ℓ.

c. Make a graph of your equation and describe its shape.

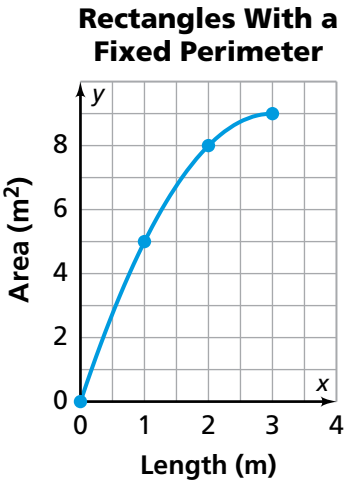
d. Use your equation to find the area of the rectangle with a length of 10 meters.

e. How could you find the area in part (d) by using your graph?

f. How could you find the area in part (d) by using a table?

g. What is the maximum area possible for a rectangle with a perimeter of 30 meters? What are the dimensions of this rectangle?

10. **a.** Copy and complete the graph to show areas for rectangles with a fixed perimeter and lengths greater than 3 meters.



b. Make a table of data for this situation.

c. What is the maximum area for a rectangle with this perimeter? What are the dimensions of this rectangle?

11. **Multiple Choice** Which equation describes the graph in Exercise 10?

A. $A = \ell(\ell - 6)$ **B.** $A = \ell(12 - \ell)$

C. $A = \ell(6 - \ell)$ **D.** $A = \ell(3 - \ell)$

12. **a.** Copy and complete the table to show areas for rectangles with a fixed perimeter and a length greater than 4 meters.

b. Make a graph of the relationship between length and area.

c. What are the dimensions of the rectangle with the maximum area?

Rectangles With a Fixed Perimeter

Length (m)	Area (m^2)
0	0
1	7
2	12
3	15
4	16
5	
6	
7	
8	

13. **Multiple Choice** Which equation describes the data in the table in Exercise 12?

F. $A = \ell(8 - \ell)$ **G.** $A = \ell(16 - \ell)$

H. $A = \ell(4 - \ell)$ **J.** $A = \ell(\ell - 8)$

14. The equation $p = d(100 - d)$ gives the monthly profit p a photographer will earn if she charges d dollars for each print.

a. Make a table and a graph for this equation.

b. Estimate the price that will produce the maximum profit. Explain.

c. How are the table and graph for this situation similar to those you made in Problem 1.1? How are they different?

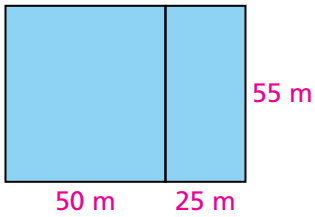
Connections

15. Of all the rectangles with whole-number side lengths and an area of 20 square centimeters, which has the least perimeter? Explain.

16. Multiple Choice What does $2(-3 + 5) + 7 \times (-4) + (-1)$ equal?

A. -55 **B.** -45 **C.** -31 **D.** -25

17. Eduardo's neighborhood association subdivided a large rectangular field into two playing fields as shown in the diagram.



a. Write expressions showing two ways you could calculate the area of the large field.

b. Use the diagram and your expressions in part (a) to explain the Distributive Property.

For Exercises 18–21, use the Distributive Property to write the expression in expanded form. Then, simplify.

18. $21(5 + 6)$ **19.** $2(35 + 1)$ **20.** $12(10 - 2)$ **21.** $9(3 + 5)$

For Exercises 22–24, use the Distributive Property to write the expression in factored form.

22. $15 + 6$ **23.** $42 + 27$ **24.** $12 + 120$

For Exercises 25 and 26, solve each equation.

25. $5x - 30 = 95$

26. $22 + 4x = 152 - 9x$

For Exercises 27–30, do the following:

- Describe the pattern of change for each function.
- Describe how the pattern of change would look in a graph and in a table. Give as many details as you can without making a graph or table.

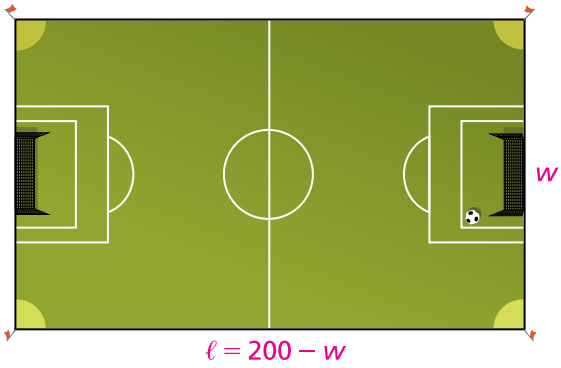
27. $y = 5x + 12$

28. $y = 10 - 3x$

29. $y = 3^x$

30. $y = \frac{15}{x}$

31. A rectangular soccer field has a perimeter of 400 yards. The equation $\ell = 200 - w$ represents the relationship between the length ℓ and width w of the field.



a. Explain why the equation is correct.

b. Is the relationship between length and width a quadratic function? Explain.

c. Suppose a field is a nonrectangular parallelogram with a perimeter of 400 yards. Is the relationship between the side lengths the same as it is for the rectangular field?

d. Suppose a field is a quadrilateral that is not a parallelogram. The perimeter of the field is 400 yards. Is the relationship between the side lengths the same as it is for the rectangular field?

32. Mr. DeAngelo is designing a school building. The music room floor will be a rectangle with an area of 1,200 square feet.

a. Make a table showing a range of possible lengths and widths for the music room floor for ten different room arrangements.

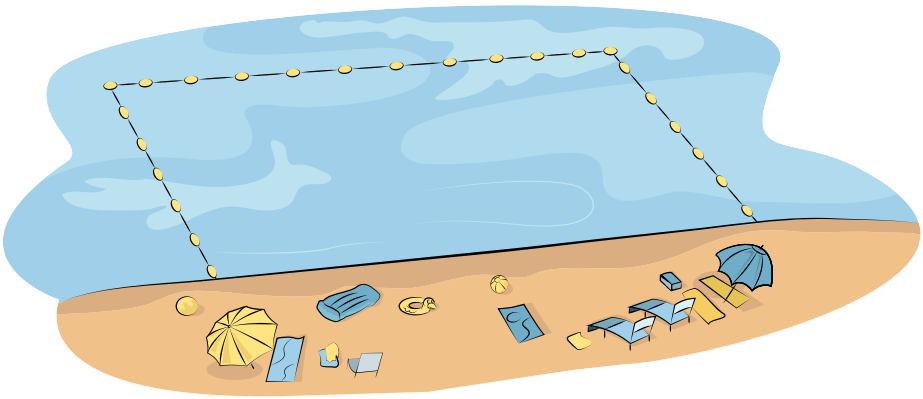
b. Add a column to your table for the perimeter of each rectangle.

c. What patterns do you see in the perimeter column? What kinds of rectangles have large perimeters? What kinds have small perimeters?

d. Write an equation you can use to calculate the length of the floor for any given width.

Extensions

33. A beach has a rectangular swimming area for toddlers. One side of the swimming area is the shore. Buoys and a rope with a length of 20 meters are used to form the other three sides.



a. How should you arrange the rope to make a rectangle with the maximum area?

b. In Problem 1.1, a fixed length of 20 meters is also used to form a rectangle. Compare the rectangle with maximum area in that Problem to the rectangle with maximum area in part (a). Are the shapes and areas of the rectangles the same? Explain.

c. Make a graph relating the length and area for the possible rectangular swimming areas. How does the graph compare with the graph from Problem 1.1?