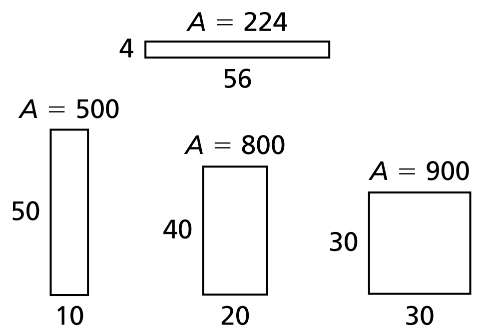
Applications

1. Students may use various sketches. Here are some examples including the rectangle with the maximum area. In general, squares will have the maximum area for a given perimeter. Long and thin rectangles will have a smaller area. This is a principle that students have encountered in earlier units of CMP, but it may be a surprising result.

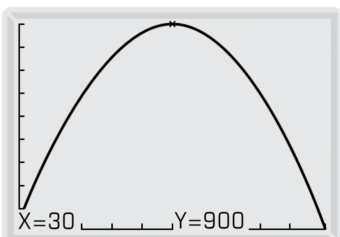


Students may use a table to verify that a square has the maximum area of 900 m^2 with side lengths of 30 m. Encourage students to take bigger increments for the base. Then students can use this to estimate where the maximum point occurs. The table can also be used to sketch a graph. Students may use the trace button on their graphing calculator to find the maximum point.

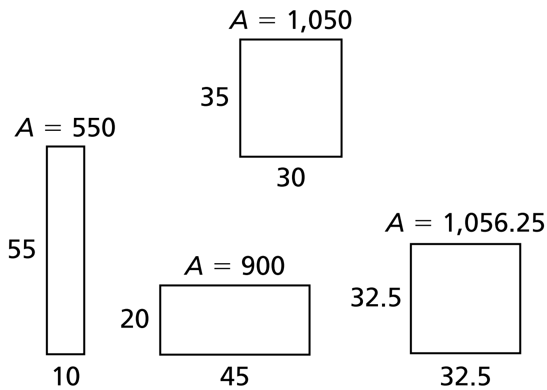
X	Y1	
0	0	
5	275	
10	500	
15	675	
20	800	
25	875	
30	900	

X=0

Students may put their sketches on graph paper to verify the areas. The rectangle with the greatest area for this fixed perimeter has sides all of which are 30.

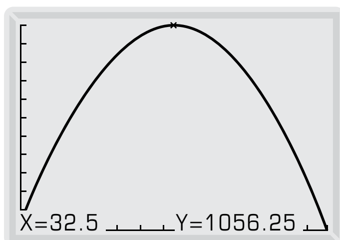


2. As in Exercise 1, sketches of possible rectangles may vary. Students may choose to consider a table or graph to analyze the situation and find that the maximum area is 1,056.25 when the sides are both 32.5. When making the table, encourage students to take bigger increments for the base. The table can be used to estimate where the maximum point occurs and sketch a graph.



X	Y1	
3	1054	
31.5	1055.3	
32	1056	
32.5	1056.3	
33	1056	
33.5	1055.3	
34	1054	

X=32.5



Note: At this point, students might not use graphing calculators, so their graphs will be sketched on paper. After Problem 1.3, the students can revisit Exercises 1 and 2 to find the equations. The equations are helpful in finding the tables or graphs. In Exercise 1, the equation is $A = \ell(60 - \ell)$, and in Exercise 2, the equation is $A = \ell(65 - \ell)$.

3. **a.** Possible answer: The graph first increases and then decreases. It has reflectional symmetry at $x = 7.5$. It crosses the x-axis at (0, 0) and (15, 0).

b. 56.25 cm^2; the rectangle has a base and width length of 7.5 cm.

c. No such minimum area rectangle exists. If we find a rectangle with a given fixed perimeter and a small area, we can always find another rectangle with the same perimeter and an even smaller area. This process of finding smaller and smaller areas can continue indefinitely. **Note:** If we propose that one of the dimensions is zero, then the area becomes zero and it is no longer a rectangle.

d. 36 cm^2

e. This can be found by using a point on the graph. For example, the point (5, 50) represents a rectangle with an area of 50 cm^2 and dimensions 5 cm by 10 cm. Because the dimensions are 5 and 10, the fixed perimeter is 30 cm. I could also look at the greatest area, y, on the graph. The greatest area of this graph is at $x = 7.5$. Because the greatest area always represents a square, the fixed perimeter is $4 \times 7.5 = 30$.

4.

Length (cm)	Width (cm)	Area (cm^2)
0	15	0
1	14	14
2	13	26
3	12	36
4	11	44
5	10	50
6	9	54
7	8	56
7.5	7.5	56.25
8	7	56
9	6	54
10	5	50
11	4	44
12	3	36
13	2	26
14	1	14
15	0	0

a. In the table, the maximum area, 56.25 cm^2, is for the side length of 7.5 cm. This point, (7.5, 56.25), is in the middle of the range of values for length in the table and is the highest point of the graph. As the length of a side increases from 7.5 to 15, the area decreases from 56.25 to 0. This increase and then decrease can be seen in the "Area" column in the table as it is shown in the shape of graph.

b. Looking for the maximum area in the table, I need to find the middle of the range of values for length in the table or where the area starts to decrease. The y-value (area) of this point is the maximum area. For any point, I divide the area (y-value) by the length (x-value) to find the width.

5. a. Possible answer: The graph first increases and then decreases. It has reflectional symmetry at $x = 25$. It crosses the x-axis at (0, 0) and (50, 0). The maximum y-value is 625.

b. 625 m^2; the length and width are both 25.

c. 400 m^2; 400 m^2; these two rectangles are related because they have the same dimensions and area, but the length and width are switched.

d. The dimensions are 20 m by 30 m

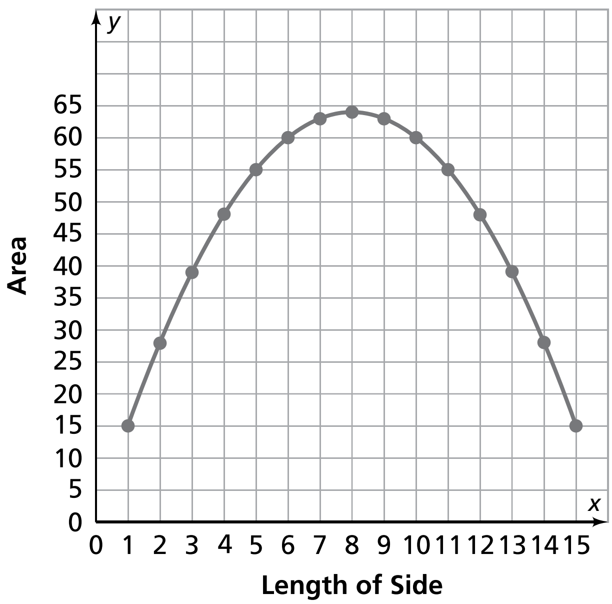
e. 100 m; if the length is 10 m, the area is 400 m^2, so the width is 40 m. So, since $P = 2(\ell + w)$, the perimeter is $2(10 + 40) = 100$ m. Students might take advantage of the observation in part (c).

6. a. As the length of a side increases by 1, the area increases first, and then it decreases after the length of a side is more than 8.

b. 32 m

c. The shape of the graph is a parabola that opens down.

Rectangles With a Perimeter of 32



d. Possible approximate dimensions: 0.75 m by 15.25 m

e. The dimensions should be 8 m by 8 m.

7. a. The graph is a parabola that crosses the x-axis at (0, 0) and (20, 0). It has a greatest point at the point (10, 100). **Note:** Students may choose to use their graphing calculators to sketch this graph and use the trace button or make a table to obtain various characteristics from the graph.

b. The highest point on the graph is at $x = 10$. The corresponding y-value is 100. For this reason, the maximum area is 100 m^2 and the dimensions are 10 m by 10 m.

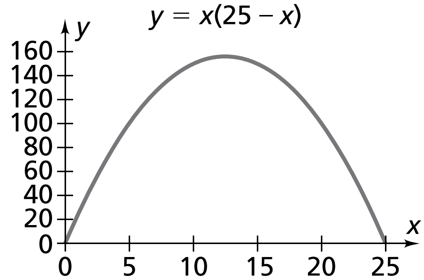
c. $A = \ell(20 - \ell)$
$A = 15(20 - 15)$
$A = 15(5) = 75$ m^2

d. Using the dimensions of the rectangle with the maximum area, the fixed perimeter $P = 2(10 + 10) = 40$. I can also find this from the equation, where 20 is the sum of the dimensions of the rectangles. Multiplying the sum by 2, the fixed perimeter 40 m.

8. a. $w = 25 - \ell$

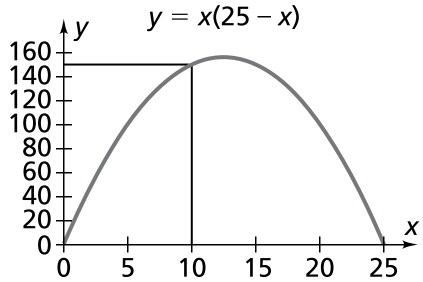
b. $A = \ell(25 - \ell)$

c. The graph goes up and then down with the shape of an upside-down U, or a parabola. It has reflectional symmetry at $x = 12.5$.



d. $A = \ell(25 - \ell)$
$A = 10(25 - 10)$
$A = 10(15) = 150$ m^2

e. First look for the length on the x-axis, and then go up until you hit the curve. From there, go across to the y-axis. At this point, the y-value or area is 150 m^2.



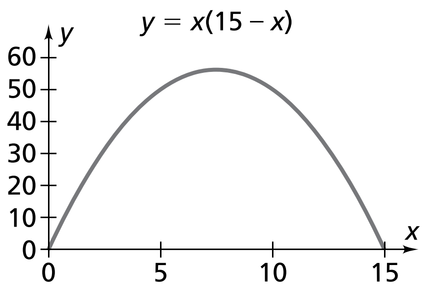
f. I go down the "Length" column until I get to 10, and then go across to find the area of 150 m^2.

g. To find the maximum area, students can either use a table, graph, or a trace on their calculator. The maximum area is 156.25 m^2 and the dimensions are 12.5 m by 12.5 m.

9. a. $15 - \ell$

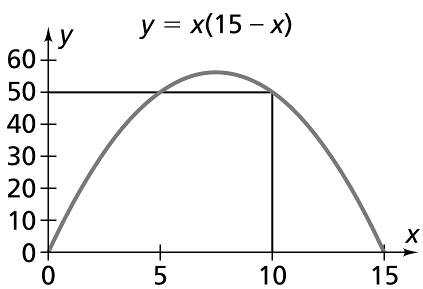
b. $A = \ell(15 - \ell)$.

c. The graph is a parabola that opens down. It has reflectional symmetry at $x = 7.5$.



d. $A = \ell(15 - \ell)$
$A = 10(15 - 10)$
$A = 10(5) = 50\ m^2$

e. First look for the length on the x-axis, and then go up until you hit the curve. Go across to the y-axis values. At this point, the y-value or area is 50 m^2.

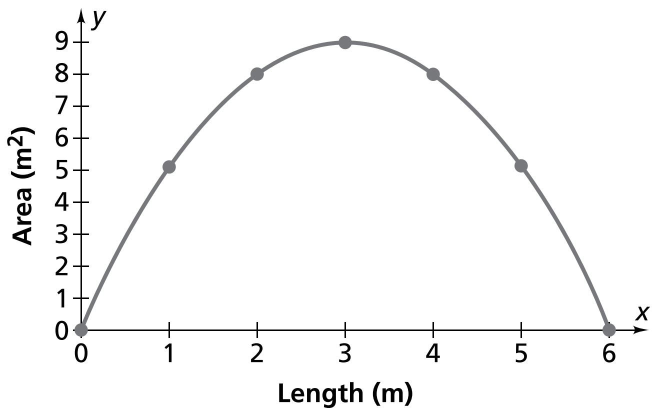


f. Go down in the "Length" column until you get to 10 m, and then go across to find the area of 50 m^2.

g. To find the maximum area, students can either use a table, graph, or trace button on their calculator. The maximum area is 56.25 m^2 with dimensions 7.5 m by 7.5 m.

10. a. Students may use symmetry to complete their graphs with the additional points and then fill in the curve.

Rectangles With a Fixed Perimeter



b. Rectangles With Perimeter of 12 m

Length (m)	0	1	2	3	4	5	6
Area (m^2)	0	5	8	9	8	5	0

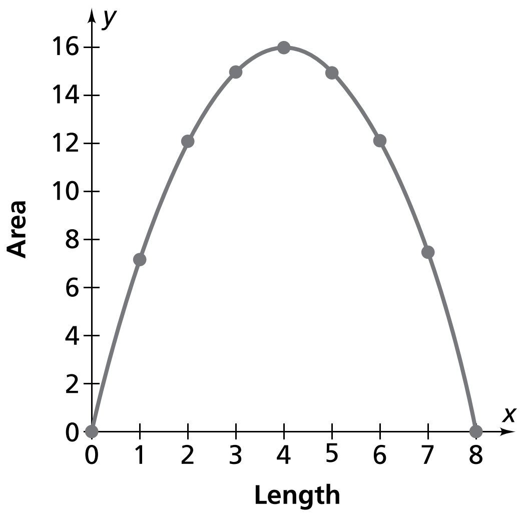
c. The maximum area is 9 m^2 with dimensions of 3 m-by-3 m.

11. C

12. a. **Rectangles With Lengths Greater Than 4**

Length (m)	Area (m^2)
0	0
1	7
2	12
3	15
4	16
5	15
6	12
7	7
8	0

b. **Rectangles With Perimeters of 16**



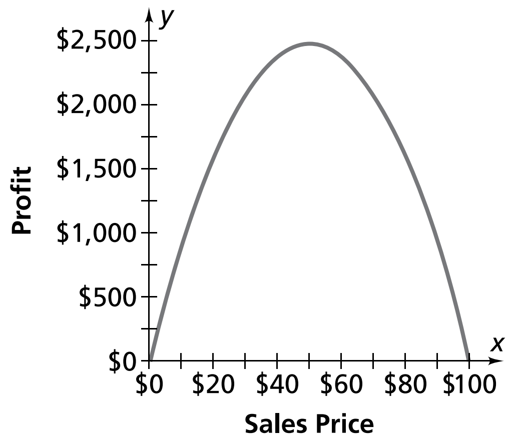
c. The dimensions are 4 m-by-4 m.

13. F

14. a. **Profits of a Photographer**

Sales Price	Profit
$0	$0
$10	$900
$20	$1,600
$30	$2,100
$40	$2,400
$50	$2,500
$60	$2,400
$70	$2,100
$80	S1,600
$90	$900
$100	$0

Photographer Profits



b. The price with the most profit is $50. The highest point in the graph is (50, 2500). In the table, the "profit" column shows maximum amount, $2,500, at the sales price of $50.

c. The shape of the graph is the same. As the *x*-value increases, the *y*-value increases at first and then decreases in both the table and the graph. The equation is also the same form, but with different numbers. In Problem 1.1 the equation was $A = \ell(10 - \ell)$, while in this Exercise, the equation has 100 instead 10.

Connections

15. The rectangle with dimensions of length 4 and 5 has the least perimeter of 18 centimeters. Student can make a table to find the least perimeter.

Rectangles With an Area of 20

Length	Width	Perimeter
1	20	42
2	10	24
4	5	18
5	4	18
10	2	24
20	1	42

16. D

17. **a.** $55(50 + 25) = 4{,}125$ m^2 or $55(50) + 55(25) = 2{,}750 + 1{,}375 = 4{,}125$ m^2.

b. The Distributive Property states that if two numbers are multiplied together and one is a sum, then the other factor can be distributed over the sum. If *a*, *b*, and *c* are numbers, then the Distributive Property states that: $a(b + c) = ab + ac$. The Distributive Property also states that if each number in a sum has a common factor, then the common factor can be factored out from each number and the sum can be written as a product. The area of a rectangle that has been subdivided into two rectangles can be calculated by multiplying the length and width of the original rectangle or by calculating the area of the smaller rectangles and adding them. **Note:** Exercises 15–32 are a review of the Distributive Property from *Accentuate the Negative*. The Distributive Property will be extended in the next Investigation to quadratic expressions.

18. $21(5 + 6) = 21(5) + 21(6) = 105 + 126 = 231$

19. $2(35 + 1) = 2(35) + 2(1) = 70 + 2 = 72$

20. $12(10 - 2) = 12(10) - 12(2) = 120 - 24 = 96$

21. $9(3 + 5) = 9(3) + 9(5) = 27 + 45 = 72$

22. $15 + 6 = 3(5 + 2)$

23. $42 + 27 = 3(14 + 9)$

24. $12 + 120 = 12(1 + 10)$ or $6(2 + 20)$ or $3(4 + 40)$ or $2(6 + 60)$

25. $x = 25$

26. $x = 10$

27. As *x* increases by one unit, *y* increases by 5 units; the graph of the equation is a straight line with a slope of 5 and a *y*-intercept of 12. In the table as *x* increases by one unit, *y* increases by 5 units.

28. As *x* increases by one unit, *y* decreases by 3 units; the graph of the equation is a straight line with a slope of -3 and *y*-intercept of 10. In the table as *x* increases by one unit, the *y*-values are decreasing by 3 units.

29. As *x* increases by one unit, *y* increases by 3 times the prior *y*-value. The graph of the equation slopes upward, and increases faster as *x* increases. In the table, as *x* increases by 1, the *y*-values are 3 times the prior difference. This is an exponential function.

30. As *x* increases by one unit, *y* decreases. The decrease in *y*-values is fast for values of *x* near to zero, and slower as *x* is further from zero. Because $\frac{15}{x}$ is valid for all values of $x \neq 0$, there is no corresponding *y*-value for $x = 0$. On the graph, the curve gets closer and closer to the *y*-axis from the left, but will never cross the *y*-axis. The same pattern appears in the table for positive and negative values of *x*.

31. **a.** If *w* represents the width of the field and the length is $\ell = 200 - w$, then the perimeter of the fields is $P = 2[(200 - w) + w] = 400$ yards, which is the perimeter given.

b. No, this is a linear relation with negative slope. As the width increases, the length decreases.

c. Yes; the lengths of opposite sides of a parallelogram are equal. The perimeter is 400 yards, so half the perimeter is 200 yards. For this reason, $\ell + w = 200$ or $\ell = 200 - w$.

d. No; a quadrilateral could have 4 sides of different lengths. For example, a trapezoid could have at least one pair of opposite sides that aren't equal in length.

32. a–b.

Rectangles With an Area of 1,200 ft^2

Length (ft)	Width (ft)	Perimeter (ft)
10	120	260
20	60	160
30	40	140
40	30	140
50	24	148
60	20	160
70	17.14	174.28
80	15	190
90	13.33	206.66
100	12	224

c. According to the table above, the column of perimeter decreases first, and then increases after the length of one side is greater than 40. The rectangles with greater difference of length and width have large perimeters. The rectangles with lesser difference of length and width have small perimeters.

d. $\ell = \frac{1{,}200}{w}$

Extensions

33. a. To obtain the maximum area of 50 m^2, the dimensions of the rectangle should be 5 m by 10 m. Students can make a table to find the maximum area. It should reflect that the perimeter of 20 m only includes three sides of the rectangle. That is, $\ell + 2w = 20$ (or $2\ell + w = 20$).

Rectangles With a Three-sided Perimeter of 20 (m)

Length	Width	Area
0	10	0
2	9	18
4	8	32
6	7	42
8	6	48
10	5	50
12	4	48
14	3	42
16	2	32
18	1	18
20	0	0

b. The shape and area of both rectangles with the maximum areas are different. In part (a), the dimensions are 5 m by 10 m with 50 m^2 of area while in Problem 1.1, the dimensions are 5 m by 5 m with 25 m^2 of area.

c. Both graphs have the same parabolic shape with a maximum point, which indicates the greatest area. However, the maximum points are different.