Answers Applications Connections Extensions

Investigation 2

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 2.1
Core 1, 50–51

Problem 2.2
Core 2–4, 8–16
Other *Applications* 5–7; *Connections* 52, 53; unassigned choices from previous problems

Problem 2.3
Core 17–28
Other *Connections* 54, 55; unassigned choices from previous problems

Problem 2.4
Core 29–39
Other *Connections* 56, *Extensions* 62–64; unassigned choices from previous problems

Problem 2.5
Core 40–42, 44–47, 57–60
Other *Applications* 43, 48, 49; *Connections* 61; *Extensions* 65; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 1 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 50, 56: *Moving Straight Ahead*; 51: *Thinking with Mathematical Models*; 52, 55: *Covering and Surrounding*; 53, 54: *Comparing and Scaling*; 57, 59, 60: *Accentuate the Negative*; 58: *Bits and Pieces II*

ACE ANSWERS 2

Applications

1. a. (Figure 8)
   b.

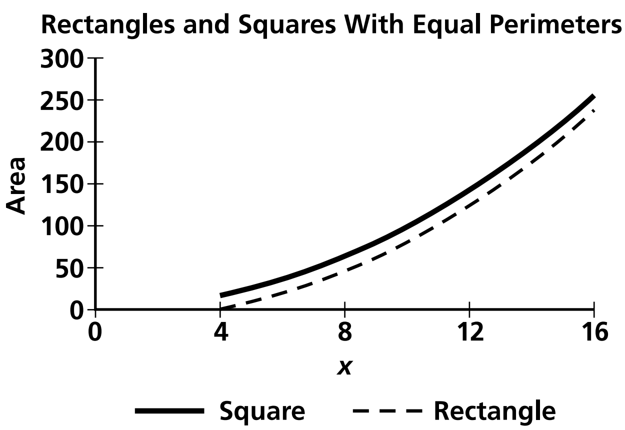


Figure 8

Square		Rectangle		
Side	Area	Length	Width	Area
4	16	8	0	0
5	25	9	1	9
6	36	10	2	20
7	49	11	3	33
8	64	12	4	48
9	81	13	5	65
10	100	14	6	84
11	121	15	7	105
12	144	16	8	128
13	169	17	9	153
14	196	18	10	180
15	225	19	11	209
16	256	20	12	240

c. The graph and the table both show that the area of the rectangle increases as the area of the square increases. The area of the square is always 16 cm^2 greater than the area of the rectangle; this constant difference between the two can be seen on the graph, but the table shows the exact value of the difference.

d. Area of the square is $A = x^2$ where x is the side length and the area of the new rectangle is $(x + 4)(x - 4)$ or $x^2 - 16$.

e. (Figures 9 and 10)

Figure 9

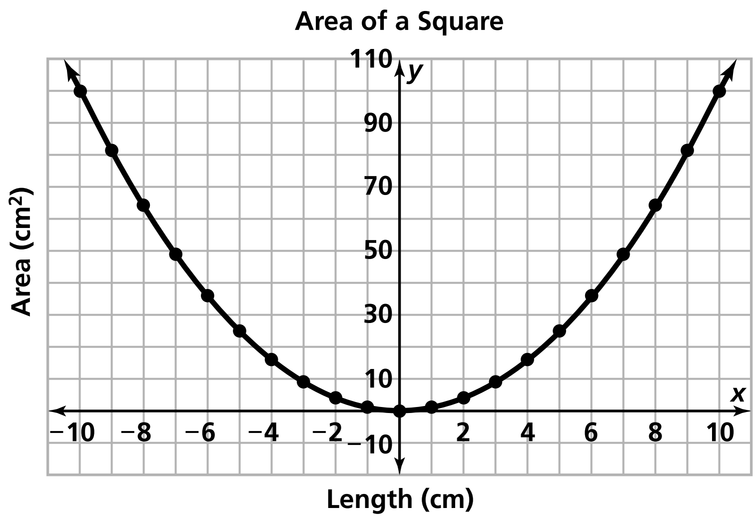
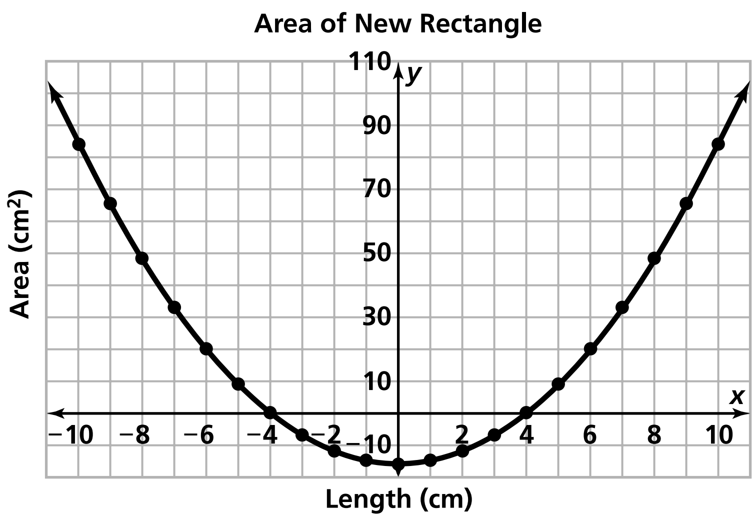
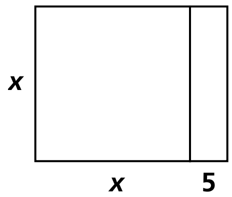


Figure 10

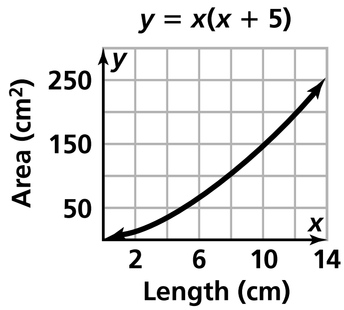


2. a.

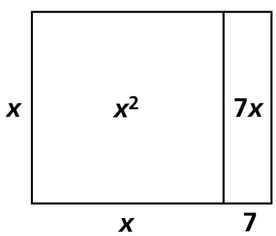


b. $A = x(x + 5)$ and $A = x^2 + 5x$

c.



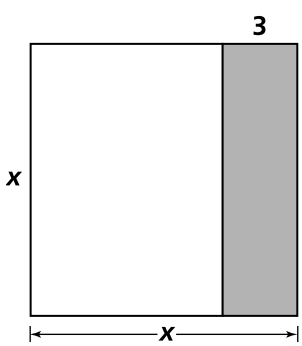
3.



$x^2 + 7x$

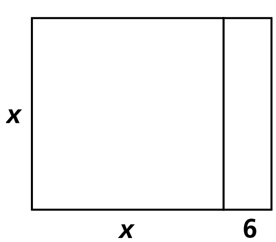
Note: In Exercises 4, 6, and 7, students may reverse the shaded/unshaded portions of the square.

4.



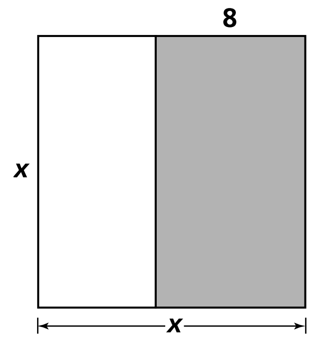
$x^2 - 3x$

5.



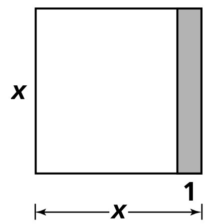
$x(x + 6)$

6.



$x(x - 8)$

7.



$x(x - 1)$

8. $x(x + 10)$

9. $x(x - 6)$

10. $x(x + 11)$

11. $x(x - 2)$

12. $x^2 + x$

13. $x^2 - 10x$

14. $x^2 + 6x$

15. $x^2 - 15x$

16. $(x + 5)x$ and $x^2 + 5x$

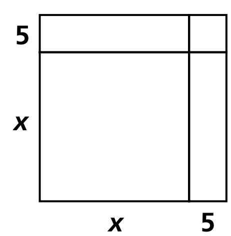
17. $x^2 + 5x + 5x + 25$ and $(x + 5)(x + 5)$

18. $x(x - 4)$ and $x^2 - 4x$

19. $(x)(2x + 3)$ and $x^2 + x^2 + 3x$

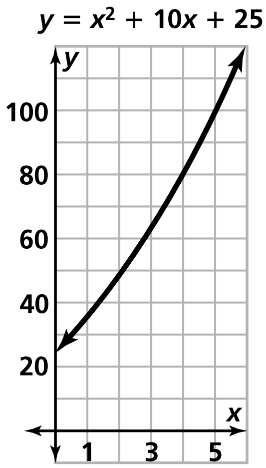
20. $(x + 5)(x + 6)$ and $x^2 + 5x + 6x + 30$

21. a.



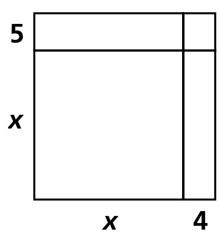
b. $(x + 5)(x + 5)$ or $x^2 + 5x + 5x + 25$ which is equivalent to $x^2 + 10x + 25$

c. $A = x^2 + 10x + 25$

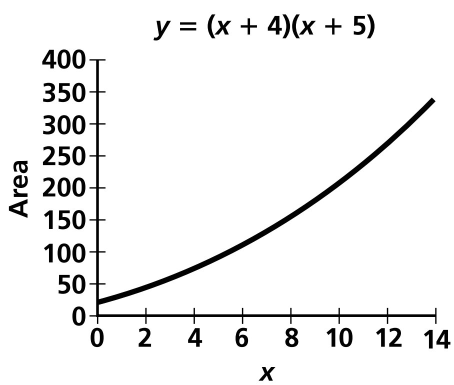


Note: To see the parabola shape we need a window which includes negative values which could not, in practical terms, represent lengths.

22. a.



b. $A = (x + 5)(x + 4)$ and
$A = x^2 + 5x + 4x + 20$



Note: To see the parabola shape we need a window which includes negative values which could not, in practical terms, represent lengths.

23. $x^2 - 3x + 4x - 12$ or $x^2 + x - 12$

24. $x^2 + 3x + 5x + 15$ or $x^2 + 8x + 15$

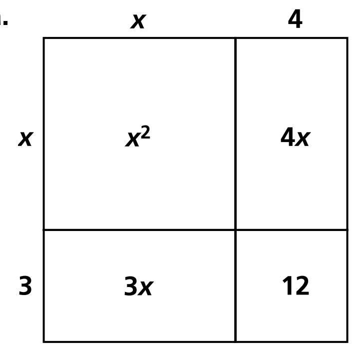
25. $x^2 + 5x$

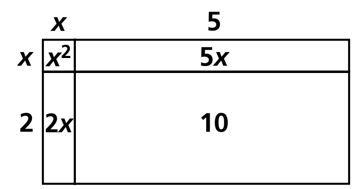
26. $x^2 - 2x - 6x + 12$ or $x^2 - 8x + 12$

27. $x^2 - 3x + 3x - 9$, or $x^2 - 9$

28. $x^2 - 3x + 5x - 15$, or $x^2 + 2x - 15$

29. a.





b. $(x + 3)(x + 4)$
$(x + 2)(x + 5)$

30. a. $(x + 12)(x + 1)$; $x^2 + 13x + 12 =$
$x^2 + 12x + 1x + 12 =$
$x(x + 12) + 1(x + 12) =$
$(x + 12)(x + 1)$

b. $(x - 12)(x - 1)$; $x^2 - 13x + 12 =$
$x^2 - 12x - 1x + 12 =$
$x(x - 12) + -1(x - 12) = (x - 12)(x - 1)$

c. $(x + 6)(x + 2)$; $x^2 + 8x + 12 =$
$x^2 + 6x + 2x + 12 =$
$x(x + 6) + 2(x + 6) = (x + 6)(x + 2)$

d. $(x - 6)(x - 2)$; $x^2 - 8x + 12 =$
$x^2 - 6x - 2x + 12 = x(x - 6) +$
$-2(x - 6) = (x - 6)(x - 2)$

e. $(x + 3)(x + 4)$; $x^2 + 7x + 12 =$
$x^2 + 3x + 4x + 12 =$
$x(x + 3) + 4(x + 3) = (x + 3)(x + 4)$

f. $(x - 3)(x - 4)$; $x^2 - 7x + 12 =$
$x^2 - 3x - 4x + 12 =$
$x(x - 3) + -4(x - 3) = (x - 3)(x - 4)$

g. $(x + 12)(x - 1)$; $x^2 + 11x - 12 =$
$x^2 + 12x - 1x - 12 =$
$x(x + 12) + -1(x + 12) = (x + 12)(x - 1)$

h. $(x - 12)(x + 1); x^2 - 11x - 12 =$
$x^2 - 12x + 1x - 12 =$
$x(x - 12) + 1(x - 12) = (x - 12)(x + 1)$

i. $(x + 6)(x - 2); x^2 + 4x - 12 =$
$x^2 + 6x - 2x - 12 =$
$x(x + 6) + -2(x + 6) = (x + 6)(x - 2)$

j. $(x - 6)(x + 2); x^2 - 4x - 12 =$
$x^2 - 6x + 2x - 12 = x(x - 6)$
$+ 2(x - 6) = (x - 6)(x + 2)$

k. $(x + 4)(x - 3); x^2 + x - 12 =$
$x^2 + 4x - 3x - 12 =$
$x(x + 4) + -3(x + 4) = (x + 4)(x - 3)$

l. $(x - 4)(x + 3); x^2 - x - 12 =$
$x^2 - 4x + 3x - 12 = x(x - 4)$
$+ 3(x - 4) = (x - 4)(x + 3)$

31. Quadratic since it has an x^2 term and this is the highest power of x.

32. Not quadratic, it is linear.

33. Quadratic; it is quadratic because it is the product of two linear factors, neither of which is constant.

34. Quadratic; it is the product of two linear factors, neither of which is constant.

35. Not quadratic, it is exponential.

36. Quadratic since it has an x^2 term and this is the highest power of x.

37. Quadratic; it is the product of two linear factors, neither of which is constant.

38. Not quadratic, it is linear.

39. Quadratic since it has an x^2 term and this is the highest power of x.

40. a. Line of symmetry: $x = 0$;
x-intercepts: 3 and -3; y-intercept: -9;
Minimum: $(0, -9)$

b. Line of symmetry: $x = -\frac{5}{2}$;
x-intercepts: 0 and -5; y-intercept: 0;
Minimum: $(-\frac{5}{2}, -\frac{25}{4})$

c. Line of symmetry: $x = -4$;
x-intercepts: -3 and -5; y-intercept: 15;
Minimum: $(-4, -1)$

d. Line of symmetry: $x = -1$;
x-intercepts: 3 and -5; y-intercept: -15;
Minimum: $(-1, -16)$

e. Line of symmetry: $x = 1$;
x-intercepts: -3 and 5; y-intercept: -15;
Minimum: $(1, -16)$

41. a. $y = (x + 3)(x + 2)$

b. y-intercept: $(0, 6)$;
x-intercepts: -3 and -2

c. Minimum: $(-2.5, -0.25)$

d. $x = -2.5$

e. The factored form can be useful in predicting the x-intercepts and the axis of symmetry. The expanded form can be useful in predicting the y-intercept. Students may have different preferences in equation forms, however they should be able to justify their choices.

42. a. $y = (x + 5)(x - 5)$

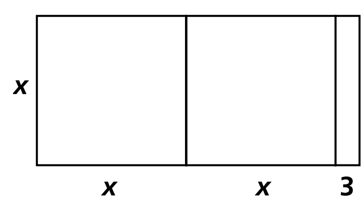
b. y-intercept: $(0, -25)$;
x-intercepts: -5 and 5

c. Minimum: $(0, -25)$

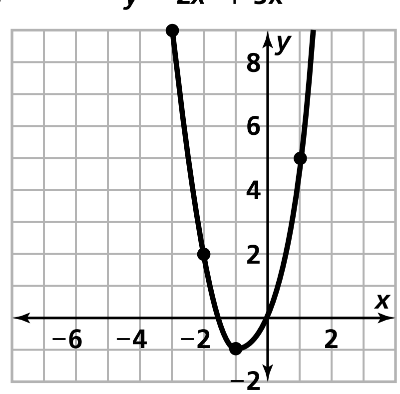
d. $x = 0$

e. The factored form can be useful in predicting the x-intercepts and the axis of symmetry. The expanded form can be useful in predicting the y-intercept. Students may have different preferences in equation forms, however they should be able to justify their choices.

43. a. Students may choose to draw a rectangle to help them answer this problem. They can represent the area as $A = x(2x + 3)$



b. $y = 2x^2 + 3x$



c. The x-intercepts are $(0, 0)$ and $(-\frac{3}{2}, 0)$. To find the x-intercept on a graph you find the point(s) where the parabola hits the x-axis. To determine the x-intercepts from the equation, find the values for x that make the factors $2x + 3$ and x equal to zero.

44–47. The line of symmetry can be found by finding the point on the x-axis that is halfway between the x-intercepts. If this point is a, then the line of symmetry is $x = a$. The x-intercepts can be read directly from the factored form or estimated from the graph of a quadratic equation.

44. a. B **b.** $x = -\frac{9}{2}$

45. a. D **b.** $x = -\frac{3}{2}$

46. a. C **b.** $x = -1$

47. a. A **b.** $x = 0$

48. a. Possible answer: They both have the same x-intercepts and they both have the same axis of symmetry.

b. Possible answer: One opens up and the other opens down.

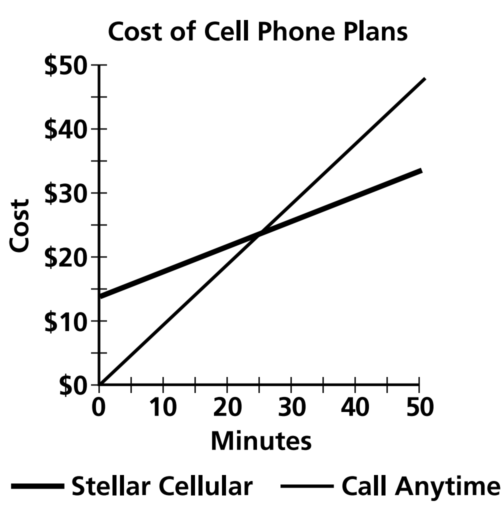
c. $(5, 25)$; the x-intercepts are $(0, 0)$ and $(10, 0)$, so the vertex is on the line $x = 5$. Substituting $x = 5$ into $y = x(10 - x)$ produces $y = 25$.

d. $(5, -25)$; substituting $x = 5$ into $y = x(x - 10)$ produces $y = -25$.

49. D

Connections

50. a. C is the cost for t minutes. Stellar Cellular: $C = 13.95 + 0.39t$, Call Anytime: $C = 0.95t$



Calls per Minute in Cell Phone Plans

Time in Minutes	Cost in Dollars	
	Stellar Cellular	Call Anytime
0	$13.95	-
5	$15.90	$4.70
10	$17.85	$9.50
15	$19.80	$14.25
20	$21.75	$19.00
25	$23.70	$23.75
30	$25.65	$28.50
35	$27.60	$33.25
40	$29.55	$38.00
45	$31.50	$42.75
50	$33.45	$47.50

b. Neither of these plans are quadratic. Both are linear. This can be seen in the rules since we do not multiply t by another factor of t. Both equations are in the linear form $y = mx + b$. In the table, you can see that both have a constant rate of change, which means they are linear. For the Stellar Cellular, the cost increases $1.95 for every 5 minutes. In the Call Anytime plan, the increase is $4.75 every 5 minutes. Both graphs look like straight lines, so they are not quadratic.

c. The plans are equal when the number of minutes is about 25 reading from the table. Solving the equation $13.95 + 0.39t = 0.95t$ for t gives an exact answer of about 24.91 minutes.

51. a. $P = \frac{\$500}{n}$

b. This is an inverse relationship: as the number of friends increases, the amount of money each person receives decreases, $n \neq 0$.

c. A graph would help you answer questions about how the amount of money each person receives changes with the number of people sharing the prize. A table would help answer questions about how much money each person would receive given a specific number of friends. An equation would help answer specific questions about any value of n.

d. This relationship is inverse, which can be seen from the graph or the equation. Students investigated inverse relationships in *Thinking With Mathematical Models.*

52. **a.** Recall, $C = \pi d$, where d is the diameter. So, $x = \pi d$. Or, we can say that $d = \frac{x}{\pi}$.

b. The radius is one half of the diameter, so radius $= \frac{1}{2}(\frac{x}{\pi})$ or $r = \frac{x}{2\pi}$.

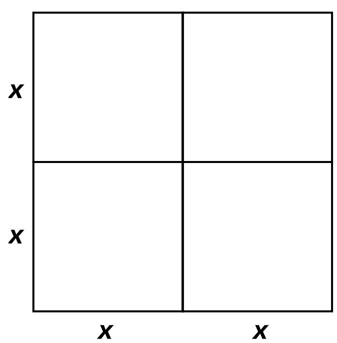
c. $A = \pi r^2$, where r is the radius; $A = \pi(\frac{x}{2\pi})^2$.

d. This is a quadratic relation since the x-value is squared.

e. $C = 10$ ft, $d = \frac{10}{\pi} \approx 3.18$ feet, $r = \frac{10}{2\pi} = \frac{5}{\pi} \approx$ 1.59 feet and $A = \pi(\frac{10}{2\pi})^2 \approx 7.96$ feet.

53. **a.** $A = 2x(2x)$ or $4x^2$

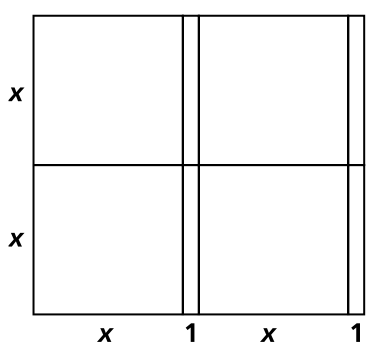
b. The area of the new square is 4 times the area of the original square. Students may choose to make a drawing to help them see this relation between the areas.



c. Yes; the angles are still 90° and the ratios of pairs of corresponding sides are 2:1.

54. **a.** $A = 2(x + 1)(2x)$ or $4x^2 + 4x$

b. The area of the new rectangle is 4 times the area of the original rectangle. It can be seen on the drawing below.



c. Yes; the angles are still 90° and the ratios of pairs of corresponding sides are 2:1.

55. **Rectangle:** $A = \ell(10 - \ell) = 10\ell - \ell^2$ and $P = \ell + \ell + (10 - \ell) + (10 - \ell) = 20$.

Parallelogram: Area cannot be determined since you are not given the height. $P = 20$.

Symmetric Kite: $P = 20$; Area cannot be determined. We can make two triangles by drawing diagonals, but we don't know the bases or heights, so comparing area is not possible.

Non-isosceles Trapezoid: Area and perimeter cannot be determined. Area cannot be determined because you are not given the length of one of the bases or the height. The perimeter cannot be determined because you are not given the length of the other two sides.

Isosceles Right Triangle: Since the triangle is isosceles right the base is $10 - \ell$ and the height is $10 - \ell$ so

$A = \frac{1}{2}(10 - \ell)(10 - \ell) = 50 - 10\ell - \frac{1}{2}\ell^2$ and

$P = \ell + (10 - \ell) + (10 - \ell) = 20 - \ell$.

56. **a.** $y = x$

b. No; given two points, there is only one line that you can draw through them.

57. If $x = 5$, then $x(x - 5) = 0$. If $x = -5$, then $x(x - 5) = 50$.

58. If $x = 1$, then $3x^2 - x = 2$. If $x = \frac{1}{3}$, then $3x^2 - x = 0$.

59. If $x = 2$, then $x^2 + 5x + 4 = 18$.
If $x = -4$, then $x^2 + 5x + 4 = 0$.

60. If $x = -2$, then $(x - 7)(x + 2) = 0$. If $x = 2$, then $(x - 7)(x + 2) = -20$.

61. **a.** The equation was $y = x(x + 4)$. The new graph would have x-intercepts at $x = -5$ and $x = -1$, and the equation would be $y = (x + 1)(x + 5)$.

b. The original equation was $y = 2x(x + 4)$. The new parabola would have x-intercepts at $x = 0$ and $x = 4$, and the equation would be $y = 2x(x - 4)$.

c. If you translate the vertex of Graph E right by 3 units it would coincide with the vertex of graph G. But the shapes of the parabolas are different. So we would need more than a translation to transform one parabola into the other.

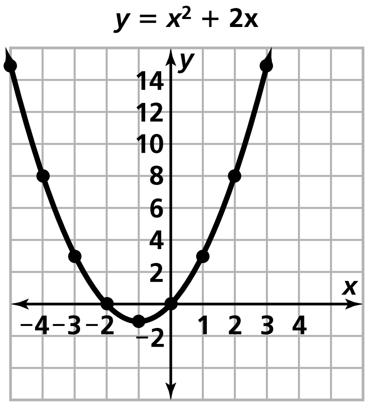
Extensions

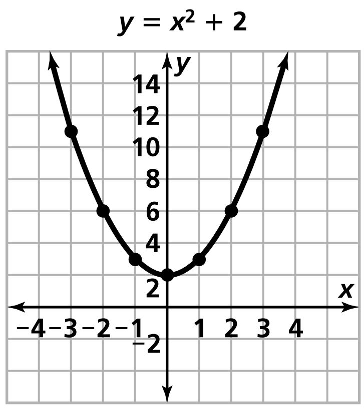
62. H

63. $(2x + 1)(x + 1)$

64. $(2x + 3)(2x + 2)$

65. a. The graphs are both parabolas, which open up.





b. The graph of $y = x^2 + 2x$ has x-intercepts of 0 and -2 and the graph of $y = x^2 + 2$ has no x-intercepts. It doesn't pass through the x-axis.

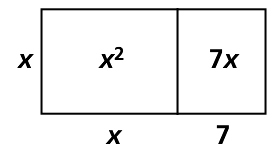
c. The y-intercept for $y = x^2 + 2x$ is (0, 0) and for $x^2 + 2$, it is (0, 2).

d. The graph of $y = x^2 + 2x$ has x-intercepts of 0 and -2. It is not possible to find the x-intercepts for the equation $y = x^2 + 2$ because there is no value of x that that you could square and add 2 and get zero.

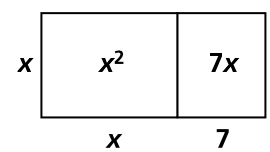
e. Yes; A parabola will always cross the y-axis since if you extend the end of the parabola out to the right and out to the left eventually it is going to cross the y-axis.

Possible Answers to Mathematical Reflections

1. Possible answer: The Distributive Property can be modeled using a rectangle. For example, if you are multiplying x and $x + 7$ you can make a rectangle model like the one below. The area of the rectangle can be represented by $x(x + 7)$ or $x^2 + 7x$. The Distributive Property states that $x(x + 7) = x^2 + 7x$ which is the same as saying that the expressions for the area of the rectangle, $x^2 + 7x$ and $x(x + 7)$, are equivalent.



2. a. To find the expanded form, draw a rectangle whose dimensions are the given factors and then subdivide the rectangle as follows:



The expanded form is the sum of the area of the parts of the rectangle $x^2 + 7x$. So $x(x + 7) = x^2 + 7x$. Each of these expressions represents the area of the above rectangle. The factored form represents the method of finding the area of the rectangle by multiplying its width and length. The expanded form represents the method of subdividing the rectangle into smaller rectangles using information from the dimensions. You can also apply the Distributive Property directly.

b. In the expanded form, $x^2 + 7x + 12$, each term represents the area of a part of the rectangle. The middle term, $7x$, is the sum of two parts. To find the factored form, find a rectangle that can be subdivided into four smaller rectangles as follows:

x^2	$4x$
$3x$	12

$\longrightarrow$

	x	4
x	x^2	$4x$
3	$3x$	12

Trial and error must be used to find the areas that add to $7x$. Different combinations can be tried and then the dimensions of the large rectangle found. Then you must check to make sure that the area of each small rectangle can be found using these dimensions. The coefficients of x in each case must add to 7 and the product of the coefficients must be 12. The Distributive Property can also be applied directly. This also involves some trial and error to write the x term, $7x$, into two parts such that the product of their coefficients is equal to the constant term 12.

$x^2 + 7x + 12 =$
$x^2 + 3x + 4x + 12 =$
$x(x + 3) + 4(x + 3) = (x + 3)(x + 4)$.

3. An equation in factored form is quadratic if it has exactly two linear factors—each factor represents a linear relation that contains the variable x. An equation in expanded form is quadratic if the highest power of the variable is 2 or it contains an x^2 term and no other term with a higher exponent. It is of the form $y = ax^2 + bx + c, a \neq 0$.

4. The graph of a quadratic equation is a parabola with a minimum point or maximum point. Parabolas are U-shaped or shaped like an upside down U. The graph has zero, one, or two x-intercepts, and it has a vertical line of symmetry that separates the graph into two symmetric parts. Every quadratic will cross through the y-axis. The y-intercept can be predicted from the equation by looking for a constant term. For example, the y-intercept for the graph of the equation $y = x^2 + 3x + 2$ is $(0, 2)$ and if there is no constant term as in the equation $y = x^2 + 2x$, the y-intercept is $(0, 0)$. The x-intercepts can be predicted easily from the factored form of the equation. For the equation $y = (x + 3)(x + 3)$, the x-intercepts are the values for x that make each factor equal to 0, so here the x-intercept is $(-3, 0)$.