Investigation 1

ACE Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 1.1

Core 1, 2, 16
Other *Connections* 14, 15; *Extensions* 30; unassigned choices from previous problems

Problem 1.2

Core 3, 5, 17–25
Other *Applications* 4, *Connections* 26, 27, 28; unassigned choices from previous problems

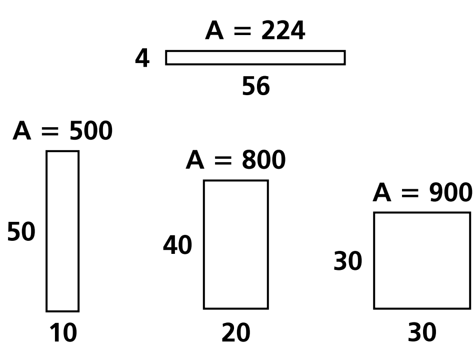
Problem 1.3

Core 6–8, 11–13
Other *Applications* 9, 10; *Connections* 29; unassigned choices from previous problems

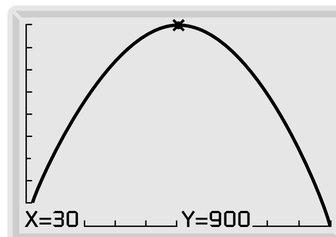
Adapted For suggestions about adapting Exercise 4 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 15, 16: *Accentuate the Negative*; 17–23: *Accentuate the Negative*; 24–28: *Moving Straight Ahead*; 29: *Thinking With Mathematical Models*

Applications

1. Students may use various sketches. Here are some examples including the rectangle with the maximum area. In general, squares will have the maximum area for a given perimeter. Long and thin rectangles will have a smaller area. This is a principle that students have encountered in earlier units of CMP, but it may yet be a surprising result.



Students may put their sketches on graph paper to verify the areas. The rectangle with the greatest area for this fixed perimeter has sides all of which are 30. **Note:** At this point, students are probably not using graphing calculators, so their graphs will be sketched on paper.



Students may use a table to verify that the maximum area of 900 is when the sides are each 30. Encourage students to take bigger increments for the base. For example, a table with increments of 5 is easy to generate. Then students can use this estimate where the maximum point occurs. The table can also be used to sketch a graph. Students may use the trace button on their graphing calculator to find the base.

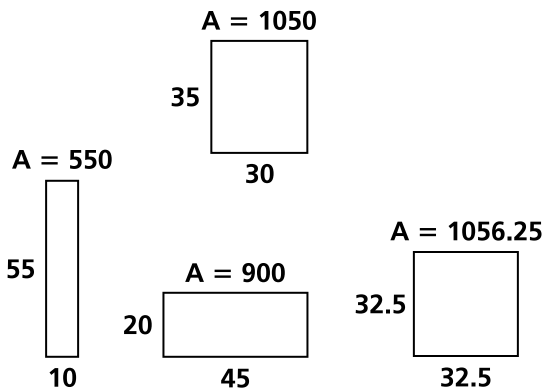
X	Y_1	
0	0	
5	275	
10	500	
15	675	
20	800	
25	875	
30	900	

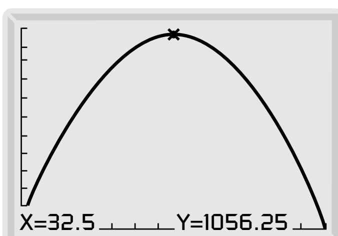
X=0

After Problem 1.3 you can come back to this problem and have the students find the equation and then connect the equation to the graph and table. The equation is: $A = \ell(60 - \ell)$. The equation is helpful in generating tables and graphs.

ACE ANSWERS 1

2. Again, sketches of possible rectangles may vary. Students may choose to consider a table or graph to analyze the situation and find that the maximum area is 1,056.25 when the sides are both 32.5. Encourage students to take bigger increments for the base. For example a table with increments of 5 is easy to generate. Then students can use this estimate where the maximum point occurs. The table can also be used to sketch a graph.





X	Y1	
3	1054	
31.5	1055.3	
32	10.56	
32.5	1056.3	
33	1056	
33.5	1055.3	
34	1054	

X=32.5

After Problem 1.3 you can come back to this problem and have the students find the equation and then connect the equation to the graph and table. The equation is: $A = \ell(65 - \ell)$. The equation is helpful in finding the table or graph.

3. a. Possible answer: The graph first increases and then decreases. It is symmetric about the line $\ell = 7.5$. It crosses the x-axis at $(0, 0)$ and $(15, 0)$.

   b. 56.25 units^2; the rectangle has a base and width length of about 7.5 units.

   c. No such minimum area rectangle exists. If we find a rectangle with a given fixed perimeter and a small area, we can always find another rectangle with the same perimeter and an even smaller area. This process of finding smaller and smaller areas can continue indefinitely. **Note:** If we propose that one of the dimensions is zero, then the area becomes zero, but then this is not a rectangle.

   d. about 36 units^2

   e. The perimeter is 30 units. This can be found by using one set of dimensions and finding the perimeter. We can use the rectangle with dimensions of 7.5 units by 7.5 units. This gives us a perimeter of 30 units. The non-zero x-intercept is 15, which is half the perimeter.

4. a. Possible answer: The graph first increases and then decreases. It is symmetric about the line $\ell = 25$. It crosses the x-axis at $(0, 0)$ and $(50, 0)$. The maximum y-value is 625.

   b. 625 m^2; the length and width are both 25

   c. 400 m^2; 400 m^2; these two rectangles are related because they have the same dimensions and area, but the length and width are switched.

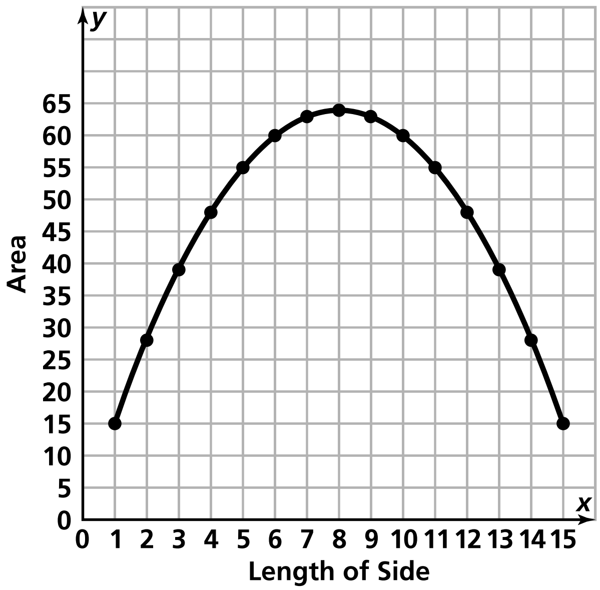
   d. 20 m by 30 m

   e. 100 m; if the length is 10 m, the area is 400 m^2, so the width is 40 m. So since $P = 2(\ell + w)$, the perimeter is $2(10 + 40) = 100$ m. Students might take advantage of the observation in part (c). For example, the area is 600 m^2 for both $\ell = 20$ and $\ell = 30$. These are the dimensions of the rectangle. Thus, the perimeter $= 2(20 + 30) = 100$ m.

5. a. As the length of a side increases by 1, the area increases first, and then it decreases after the length of a side is more than 8.

   b. 32 m. You can find the length of the other side by the equation $A = \ell w$ for the rectangular area, then add the lengths of two sides together and times it by 2.

c. The shape of the graph is a parabola that opens down.



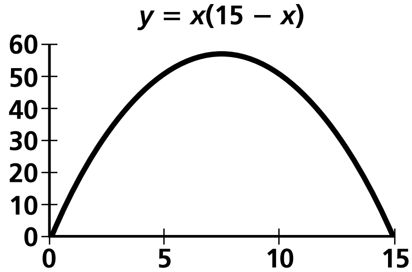
d. Possible approximate dimensions: 0.75 by 15.25

e. 8 by 8. From a table, students can find the largest value in the column of area, and then get the length of one side. From a graph, students can trace the graph to get the length of one side from the highest point with respect to area. To get the length of the other side, use the equation for rectangular area.

6. a. The perimeter of any figure is the distance around it. The perimeter of a rectangle: $P = \ell + \ell + w + w$, or $P = 2\ell + 2w$. If the perimeter of a rectangle is 30 m, and the length of one side is ℓ, then $30 = 2\ell + 2w$ or $15 = \ell + w$. So, $w = 15 - \ell$.

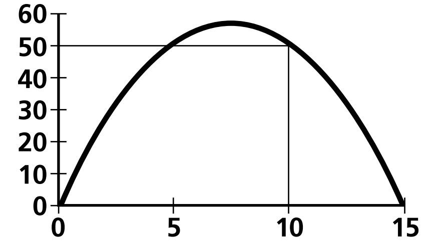
b. With the values given above $A = \ell(15 - \ell)$.

c. The graph is a parabola that opens down. It is symmetric about $x = 7.5$.



d. $A = \ell(15 - \ell)$
$A = 10(15 - 10)$
$A = 10(5) = 50 \text{ m}^2$

e. First look for the length on the x-axis, and then go up until you hit the curve. Go across to the y-axis values. This will tell you your y value or area of 50 m^2.



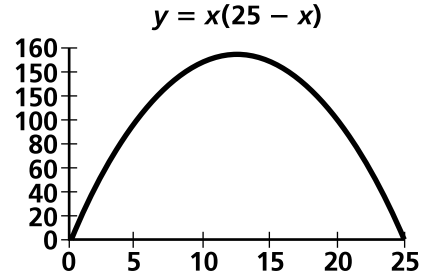
f. Go down in the column until you get to a length of 10, and then go across to find the area of 50 m^2.

g. To find the maximum area, students can either use a table, graph, or a trace on their calculator. The maximum value is when the length of one side is 7.5 m. This gives an area of 56.25 m^2.

7. a. If the perimeter of a rectangle is 50 m, and the length is ℓ, then $50 = 2\ell + 2w$ or $25 = \ell + w$, so $w = 25 - \ell$.

b. Area is the length times width, or $A = \ell w$. With the values given above $A = \ell(25 - \ell)$

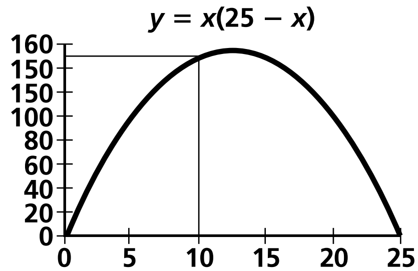
c. The graph goes up and then down, like an upside-down U, or a parabola. It is symmetric about the line $x = 12.5$.



d. $A = \ell(25 - \ell)$
$A = 10(25 - 10)$
$A = 10(15) = 150 \text{ m}^2$

ACE ANSWERS 1

e. First look for the length on the x-axis, and then go up until you hit the curve. From there, go across to the y-axis. This will tell you your y-value or area of 150 m^2.



f. Go down in the column until you get to a length of 10, and then go across to find the area of 150 m^2.

g. To find the maximum area, students can either use a table, graph, or a trace on their calculator. The maximum value is when the length of one side is 12.5 m. This gives an area of 156.25 m^2.

8. a. Students may choose to use their graphing calculators to sketch this graph and either the trace function or table function to obtain various characteristics from the graph. The graph is a parabola that crosses the x-axis at (0, 0) and (20, 0). It has a greatest point at the point (10, 100).

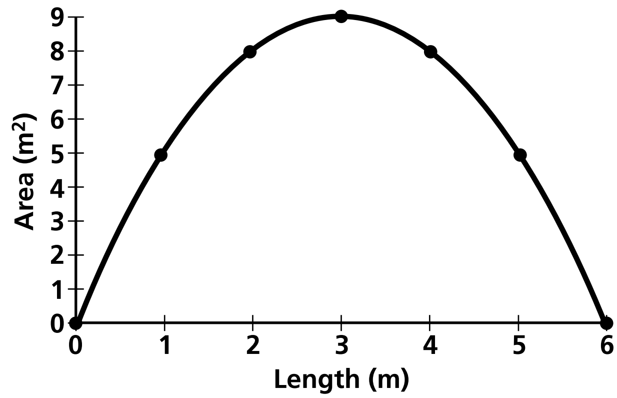
b. The dimensions of the rectangle with the greatest area is at the maximum value on the graph. These dimensions are 10 m by 10 m. The area would be 100 m^2.

c. A rectangle with length of 15 m would have an area of: $A = 15(20 - 15)$, $A = 15(5)$, or $A = 75$ m^2.

d. The fixed perimeter would be 40 units. You can find this by finding the perimeter of the maximum area rectangle, 10 units by 10 units, or you can find this from the equation, where 20 is the sum of two sides of the rectangles.

9. a. Students may use symmetry to complete their graphs with the additional points and then fill in the curve.

Rectangles With a Certain Fixed Perimeter



b.

Rectangles With Perimeter of 6 m

Length (m)	0	1	2	3	4	5	6
Area (m^2)	0	5	8	9	8	5	0

c. The rectangle with the greatest area has dimensions of 3 m by 3 m. Students may look for the maximum point on the graph and find that the maximum occurs when the length is 3 m.

10. C

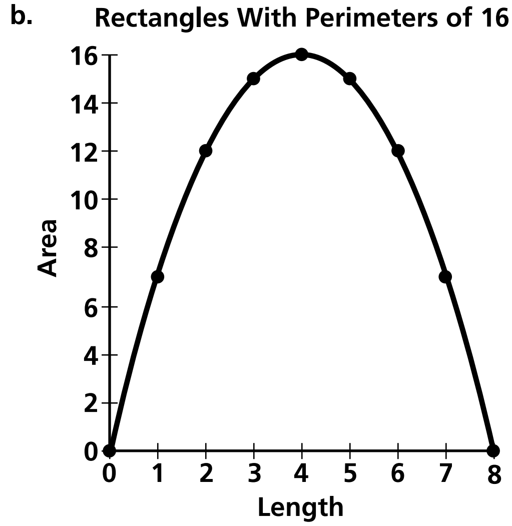
11. a.

Rectangles With Lengths Greater Than 4

Length (m)	Area (m^2)
0	0
1	7
2	12
3	15
4	16
5	15
6	12
7	7
8	0

By applying the equation $A = \ell w$, you can find the values and the pattern of changes in width and then complete the table.

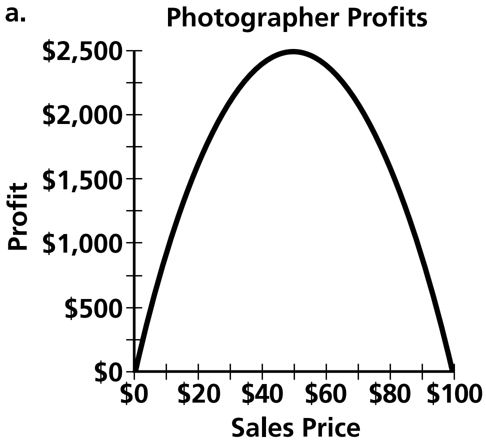
b.



c. The greatest area rectangle has dimensions 4 m by 4 m.

12. F; $A = \ell(8 - \ell)$

13. a.



Profits of a Photographer

Sales Price	Profit
$0	$0
$10	$900
$20	$1,600
$30	$2,100
$40	$2,400
$50	$2,500
$60	$2,400
$70	$2,100
$80	$1,600
$90	$900
$100	$0

b. You can either get this from the table or graph. The price with the most profit is $50.

c. The shape of the graph seems to be the same. As the x-value increases, the y-value increases at first and then decreases in both the table and the graph.

The equation seems to be taking the same form. The only thing that seems to change is the number in the equation. In Problem 1.1 the equation was $A = \ell(10 - \ell)$, while in this problem, the equation had a 100 instead of a 10.

Connections

14. The rectangle with sides of length 4 and 5 has the smallest perimeter of 18 centimeters. You can use a table like the one below with ℓ, w, and P to find the smallest perimeter.

Rectangles With an Area of 20

Length	Width	Perimeter
1	20	42
2	10	24
4	5	18
5	4	18
10	2	24
20	1	42

15. D

16. a. 4,125 square meters; The area is $55(50 + 25)$ or $55(50) + 55(25) = 2{,}750 + 1{,}375 = 4{,}125$ square meters.

b. The Distributive Property states that if two numbers are multiplied together and one is a sum, then the other factor can be distributed over the sum. That is, the factor is multiplied by each number in the sum. If a, b, and c are numbers, then the Distributive Property states that: $a(b + c) = ab + ac$. The Distributive Property also states that if each number in a sum has a common factor, then the common factor can be factored out from each number and the sum can be written as a product. The area of a rectangle that has been subdivided into two rectangles can be

ACE ANSWERS 1

calculated by multiplying the length and width of the original rectangle or by calculating the area of the smaller rectangles and adding them.

Note that Exercises 15–32 are a review of the Distributive Property from *Accentuate the Negative*. The Distributive Property will be extended in the next investigation to quadratic expressions.

17. $21(5 + 6) = 21(5) + 21(6) = 105 + 126 = 231$

18. $2(35 + 1) = 2(35) + 2(1) = 70 + 2 = 72$

19. $12(10 - 2) = 12(10) - 12(2) = 120 - 24 = 96$

20. $9(3 + 5) = 9(3) + 9(5) = 27 + 45 = 72$

21. $15 + 6 = 3(5 + 2)$

22. $42 + 27 = 3(14 + 9)$

23. $12 + 120 = 12(1 + 10)$ or $6(2 + 20)$ or $3(4 + 40)$ or $2(6 + 60)$

24. $x = 25$

25. $x = 10$

26. As x increases by one unit, y increases by 5 units; the graph of the equation is a straight line with a slope of 5 and a y-intercept of 12. In the table as x increases by one unit, y increases by 5 units.

27. As x increases by one unit, y decreases by 3 units; the graph of the equation is a straight line with a slope of -3 and y-intercept of 10. In the table as x increases by one unit, the y-values are decreasing by 3 each time.

28. **a.** If w represents the width of the field and if the length is $\ell = 150 - w$ then the perimeter of the fields is $P = (150 - w) + (150 - w) + w + w = 300$ yards which is the perimeter given. So the equation of length works.

b. This is a linear relation with negative slope. As the width increases the length decreases.

c. Yes. The fact that the lengths of opposite sides of a parallelogram are equal guarantees the correctness of that equation. Since the perimeter is 300 yards, half the perimeter is 150 yards. You can find the perimeter by taking $2\ell + 2w$. Since the opposite sides of a parallelogram are of equal length, half the perimeter is $\ell + w$. If the width is represented by w, the length is the rest of the 150 yards or $150 - w$.

d. No. The quadrilateral doesn't guarantee equal length of opposite sides. That is, the length of each side of a quadrilateral could be different from each other for example a trapezoid could have at least one pair of opposite sides that aren't equal in length.

29. **a, b.**

Rectangles With an Area of 1,200 (ft^2)

Length (ft)	Width (ft)	Perimeter (ft)
10	120	260
20	60	160
30	40	140
40	30	140
50	24	148
60	20	160
70	17.14	174.28
80	15	190
90	13.33	206.66
100	12	224

c. According to the table above, the column of perimeter decreases first, and then increases after the length of one side is greater than 40. The rectangles with smaller difference between length and width have small perimeters. The rectangles with larger difference between length and width have large perimeters.

d. $\ell = \dfrac{1{,}200}{w}$

Extensions

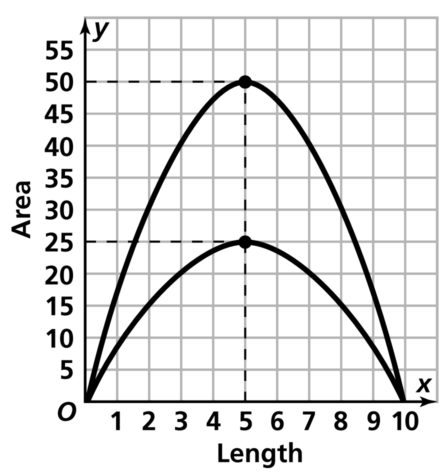
30. a. The maximum area is 50 square meters with two sides of length 5 and 10 meters. You can use a table with Length, Width, and Area to find the maximum area. Be careful: 20 meters is only for three sides of a rectangle. That is, $\ell + 2w = 20$ (or $2\ell + w = 20$). (See table below.)

Rectangles With a Three-sided Perimeter of 20 (m)

Length	Width	Area
0	10	0
2	9	18
4	8	32
6	7	42
8	6	48
10	5	50
12	4	48
14	3	42
16	2	32
18	1	18
20	0	0

b. The shape and area of both rectangles that have the maximum areas are different from each other. One has dimensions 5 by 10 with 50 square meters, while the other has dimensions 5 by 5 with 25 square meters.

c. Both graphs have the same shape of upside down U with a maximum point, where the area is the greatest. However, both graphs have different maximum points.



Possible Answers to Mathematical Reflections

1. a. There are several patterns students may recognize:

- The area increases fast at the beginning and then slows down at the maximum point and then decreases—slowly at first and then larger decreases occur as the length approaches half the perimeter.
- The x-coordinate of the maximum point occurs halfway between the x-intercepts. This value is also one-fourth of the perimeter.
- The x-intercepts occur at 0 and when x = half the perimeter.
- The graph is symmetric about a line that passes through the maximum point and is perpendicular to the x-axis.

b. In the table, the growth pattern is similar to that described for the graph. The length increases at a steady rate from 0 to a point which is half the perimeter. The area increases until it reaches a maximum point and then it decreases. The area reaches its maximum value when the length is halfway between 0 and half the perimeter, or just one-fourth of the perimeter.

2. To find the maximum area for rectangles with a fixed perimeter, you could generate a table or a graph for the situation and find the greatest entry in the "Area" column of the table or find the y-coordinate of the maximum point on the graph. Or, since a square will have the greatest area, you could just find the area for the square with this perimeter.

3. The tables of quadratic functions cannot be generated by adding or multiplying the previous entry by a constant amount. The graphs of quadratic functions are not straight lines nor do they have the shape of an exponential function. Quadratic equations differ from exponential equations in that they don't contain a variable exponent, and they differ from linear equations in that they contain two linear factors multiplied together.