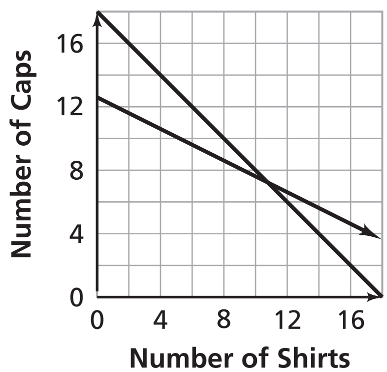
ACE

Assignment Guide for Problem 2.1

Applications: 1–17 | Connections: 33–51
Extensions: 75–77

Answers to Problem 2.1

A. **1.** Nyla's graph might look like the one below. Her reasoning is correct. However, the exact solution is difficult to find on the graph. We can only get an estimate of the solution from the graph.



Note: This graph suggests that the solution is approximately (11, 7). Because of the scale of the axes, the point of intersection is not perfectly clear. You might use this graph to motivate your students to look forward to the algebraic methods in Question B to find exact answers.

2. Jimfa's reasoning is correct. If you solve his equation for s, then $s = 11$. 11 shirts and 7 caps were sold.

B. **1.** $x = 4.5$, $y = 6.35$ **2.** $x = -1$, $y = 4$

3. $x = 11$, $y = 3$ **4.** $x = 3$, $y = 2$

5. Writing the equations of this system in equivalent $y = mx + b$ form produces the same equation in each case, $y = x + 5$. Thus every solution of one equation is a solution of the other equation, meaning that there are infinitely many solutions for the system. If one follows the "set the two expressions for y equal to each other" strategy, the result is $x + 5 = x + 5$. If you operate routinely with the strategies that are generally used to solve linear equations, you get $x = x$ or $0 = 0$, which is true for all x.

6. Writing the equations of this system in equivalent $y = mx + b$ form gives $y = x + 5$ and $y = x + 4$. Proceeding with the "set the two expressions for y equal to each other" strategy, you get $x + 5 = x + 4$ or eventually $1 = 0$. This cannot be true, regardless of the value of x, so the system must have no solution.

Note: The results of Question B, parts (5) and (6) highlight the logic of algebraic equation solving. For example, suppose that the task is to solve an equation like $4x + 3 = 6x - 5$. When we operate on both sides of the equation, what we are really saying is, "Suppose that there is a value of x that satisfies the equation." Then, $4x + 3 + 5 = 6x - 5 + 5$ and $4x + 8 = 6x$. Next, $-4x + 4x + 8 = -4x + 6x$ and $8 = 2x$. Finally, $\frac{8}{2} = \frac{2x}{2}$ and $4 = x$. So, if there is a solution, it must be $x = 4$. Checking $4(4) + 3 = 6(4) - 5$. If this reasoning leads to a contradictory result (like $1 = 0$), then our initial assumption that there is an x that satisfies the equation must be false.

C. Ming's suggestion, solving for x, has one less step than Eun Mi's suggestion.

The idea is correct and algebraically efficient. The equivalent system is

$$\begin{cases} x = 3 - y \\ x = -5 + y \end{cases}.$$

The solution comes from solving $3 - y = -5 + y$, which is $y = 4$. Then, $x = 3 - 4 = -1$. The solution is the same as Question B, part (2).